Chapter 10

EMPIRICAL APPLICATIONS OF GMM

GMM estimation has been frequently applied to rational expectations models. This chapter discusses examples of these applications. The main purpose is not to provide a survey of the literature but to illustrate applications. Problems that researchers have encountered in applying GMM are discussed as well as procedures they have used to address these problems. In this chapter, the notation for the NLIV model of Section 9.2 will be used.

10.1 Euler Equation Approach

Hansen and Richard (1987) show that virtually all asset pricing models can be written as

$$v_t = E[m_{t+1}d_{t+1}|\mathrm{I}_t] \tag{10.1}$$

where v_t is the asset price at date t, m_{t+1} is the intertemporal marginal rate of substitution (IMRS) between date t and date t+1, and d_{t+1} is the payoff of an asset at date t+1. Each asset pricing model specifies a different IMRS.

Hansen and Singleton (1982) specify the IMRS by

$$m_{t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\alpha} \tag{10.2}$$

and measure c_t by real nondurable consumption expenditures or real nondurable and service consumption expenditures. Hansen and Singleton (1984) find that the chi-square test for the overidentifying restrictions rejects their model especially when nominal risk free bond returns and stock returns are used simultaneously.[1] Their finding is consistent with the Mehra and Prescott's (1985) equity premium puzzle. When the model is rejected, the chi-square test statistic does not provide much guidance as to what causes the rejection. Hansen and Jagannathan (1991) develop a diagnostic that could provide such guidance.

Brown and Gibbons (1985) use the same specification of the IMRS but propose to measure it from asset returns data rather than consumption data. An advantage of this measurement is that asset returns data are measured without measurement errors and are free from the time aggregation problem in contrast to consumption data.

They assume that $E(\frac{c_{t+1}}{c_t}|\mathrm{I}_t)$ is a constant that does not depend on I_t. For example, this assumption is satisfied if consumption is a martingale, in which case $E(\frac{c_{t+1}}{c_t}|\mathrm{I}_t) = 1$. Then $E(\frac{c_{t+\tau}}{c_t}|\mathrm{I}_t) = E[(\frac{c_{t+\tau}}{c_{t+\tau-1}})(\frac{c_{t+\tau-1}}{c_{t+\tau-2}})\cdots(\frac{c_{t+1}}{c_t})|\mathrm{I}_t)$ is a constant that does not depend on I_t. Therefore, $E[\beta^\tau(\frac{c_{t+\tau}}{c_t})^{-\alpha}(\frac{c_{t+\tau}}{c_t})|\mathrm{I}_t] = k_\tau$ is a constant that does not depend on I_t.

Now consider a security that pays off $c_{t+\tau}$ as its payoff for $\tau = 1, 2, 3, \cdots$. Then

[1] Cochrane (1989) points out that the utility that the representative consumer loses by deviating from the optimal consumption path is very small in the Hansen-Singleton model and in the Hall's (1978) model. In this sense, the Hansen-Singleton test and Hall's test may be too sensitive to economically small deviations caused by small costs of information and transactions.

the price of the security at date t will be

$$v_t = E[\sum_{\tau=1}^{\infty} \beta^{\tau} (\frac{c_{t+\tau}}{c_t})^{-\alpha} c_{t+\tau} | \mathrm{I}_t] = (\sum_{\tau=1}^{\infty} k_{\tau}) c_t. \tag{10.3}$$

Hence, the gross rate of return from holding this security from date t to date t+1, R^m_{t+1}, is

$$R^m_{t+1} = \frac{v_{t+1} + c_{t+1}}{v_t} = k \frac{c_{t+1}}{c_t} \tag{10.4}$$

where $k = (1 + \sum_{\tau=1}^{\infty} k_{\tau}) / \sum_{\tau=1}^{\infty} k_{\tau}$. Hence the IMRS can be measured by R^m_{t+1}:

$$m_{t+1} = \beta (\frac{c_{t+1}}{c_t})^{-\alpha} = \beta^* (R^m_{t+1})^{-\alpha}. \tag{10.5}$$

where $\beta^* = \beta k^{\alpha}$. The Euler equation is

$$E(\beta^* (R^m_{t+1})^{-\alpha} R_{t+1} | \mathrm{I}_t) = 1 \tag{10.6}$$

for any asset return R_{t+1}. To apply GMM, let $\mathbf{b} = (\beta^*, \alpha)'$, $\mathbf{x}_t = (R^m_{t+1}, R_{t+1})'$, and $g(\mathbf{x}_t, \mathbf{b}) = \beta^* (R^m_{t+1})^{-\alpha} R_{t+1} - 1$ in the notation for the NLIV model.

Brown and Gibbons (1985) measure R^m_{t+1} by the New York Stock Exchange value weighted return. Even though the value weighted return is precisely measured, it is not exactly equal to R^m_{t+1} in the model because the value weighted average of the New York Stock Exchange stocks does not pay aggregate consumption as its payoff. This problem is closely related to the Roll's (1977) critique for tests of Capital Asset Pricing Models which use the value weighted returns as the market return.

Even though the Euler equation holds for any asset return, the identification assumption for GMM fails to hold when we choose R_{t+1} in (10.6) to be R^m_{t+1}. With this choice, $g(\mathbf{x}_t, \mathbf{b}) = 0$ when $\beta^* = 1$ and $\alpha = 1$.

10.2 Habit Formation and Durability

Many researchers have considered the effects of time-nonseparability in preferences on asset pricing. Let us replace (9.2) by

$$U(c_t, c_{t-1}, c_{t-2}, \cdots) = \frac{1}{1-\alpha}(s_t^{1-\alpha} - 1), \tag{10.7}$$

where s_t is the service flow from consumption purchases. Purchases of consumption and service flows are related by

$$s_t = a_0 c_t + a_1 c_{t-1} + a_2 c_{t-2} + \cdots . \tag{10.8}$$

Depending on the values of the a_τ's, the model (10.7) leads to a model with habit formation and/or durability. For example, this type of specification for time-nonseparability has been used to model durability by Mankiw (1985), Hayashi (1982), Dunn and Singleton (1986), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1990), and Ogaki and Reinhart (1998a,b), and used to model habit formation by Ferson and Constantinides (1991), Ferson and Harvey (1992), Cooley and Ogaki (1996), and Ogaki and Park (1997).[2] Heaton (1993, 1995) used it to model a combination of durability and habit formation. Constantinides (1990) argues that habit formation could help solve the equity premium puzzle. He shows how the intertemporal elasticity of substitution and the relative risk aversion coefficient depend on the parameters a_τ and α in a habit formation model.

In this section, we discuss applications by Ferson and Constantinides (1991), Cooley and Ogaki (1996), and Ogaki and Park (1997) to illustrate econometric formulations for habit formation models. We will discuss more about applications for

[2] These papers found evidence in favor of habit formation with aggregate consumption data, but Dynan (2000) finds no evidence for habit formation in household level panel data for food.

durable goods in later sections. In their models, it is assumed that $a_\tau = 0$ for $\tau \geq 2$. Let us normalize a_0 to be one, so that $\mathbf{b} = (\beta, \alpha, a_1)'$. The asset pricing equation takes the form

$$\frac{E[\beta(s_{t+1}^{-\alpha} + \beta a_1 s_{t+2}^{-\alpha})R_{t+1}|\mathrm{I}_t]}{E[s_t^{-\alpha} + \beta a_1 s_{t+1}^{-\alpha}|\mathrm{I}_t]} = 1. \tag{10.9}$$

Then let $\epsilon_t^0 = \beta(s_{t+1}^{-\alpha} + \beta a_1 s_{t+2}^{-\alpha})R_{t+1} - (s_t^{-\alpha} + \beta a_1 s_{t+1}^{-\alpha})$. Though Euler equation (10.9) implies that $E(\epsilon_t^0|\mathrm{I}_t) = 0$, this property cannot be used as the disturbance for GMM because both of the two regularity assumptions discussed in Section 9.3 are violated. These violations are caused by the nonstationarity of c_t and by the three sets of trivial solutions, $\alpha = 0$ and $1 + \beta a_1 = 0$; $\beta = 0$ and $\alpha = \infty$; and $\beta = 0$ and $a_1 = \infty$ with $\alpha > 0$. Ferson and Constantinides (1991) solve both of these problems by defining $\epsilon_t = \frac{\epsilon_t^0}{s_t^{-\alpha}}$. Since $s_t^{-\alpha}$ is in I_t, $E(\epsilon_t|\mathrm{I}_t) = 0$. The disturbance is a function of $\frac{s_{t+\tau}}{s_t}$ $(\tau = 1, 2)$ and R_{t+1}. When $\frac{c_{t+1}}{c_t}$ and R_t are assumed to be stationary, $\frac{s_{t+\tau}}{s_t}$ and the disturbance can be written as a function of stationary variables.

One problem that researchers have encountered in these applications is that $c_{t+1} + a_1 c_t$ may be negative when a_1 is close to minus one. In a nonlinear search for $\mathbf{b}_T$ or in calculating numerical derivatives, a GMM computer program will stall if it tries a value of a_1 that makes $c_{t+1} + a_1 c_t$ negative for any t. Atkeson and Ogaki (1996) have encountered similar problems in estimating fixed subsistence levels from panel data. One way to avoid this problem is to program the function $f(\mathbf{x}_t, \mathbf{b})$, so that the program returns very large numbers as the values of $f(\mathbf{x}_t, \mathbf{b})$ when non-admissible parameter values are used. However, it is necessary to ignore these large values of $f(\mathbf{x}_t, \mathbf{b})$ when calculating numerical derivatives. This process can be done by suitably modifying programs that calculate numerical derivatives.[3]

[3] A GMM User Guide (see Ogaki, 1993b) explains these modifications for Hansen/Heaton/Ogaki

The model presented in this section is the linear specification of habit formation. More recent theoretical work often adopts the nonlinear specification of habit formation as in Campbell and Cochrane (1999, 2000) and Menzly, Santos, and Veronesi (2004), among others. The model presented in this section is also a model of internal habit formation. In models of external habit formation, the habit depends on the consumption of some exterior reference group. In the Abel's (1990) model of catching up with Jones, the habit depends on per capita aggregate consumption. Campbell and Cochrane (1999, 2000), Li (2001), and Menzly, Santos, and Veronesi (2004) study models of external habit formation. Chen and Ludvigson (2004) use the sieve minimum distance estimator developed by Newey and Powell (2003) and Ai and Chen (2003) for approximating an unknown function to empirically evaluate various specifications of habit including linear/nonlinear and internal/external habit formation.The sieve minimum distance estimator is implemented in the GMM framework.

10.3 State-Nonseparable Preferences

Epstein and Zin (1991) estimate a model with state-nonseparable preference specification in which the life-time utility level v_t at period t is defined recursively by

$$V_t = \{c_t^{1-\alpha} + \beta E[V_{t+1}^{1-\alpha}|\mathrm{I}_t]\}^{\frac{1-\rho}{1-\alpha}}, \tag{10.10}$$

where $\alpha > 0$ and $\rho > 0$. The asset pricing equation for this model is

$$E[\beta^*(R_{t+1}^m)^\eta(\frac{c_{t+1}}{c_t})^\theta R_{t+1}] = 1, \tag{10.11}$$

for any asset return R_{t+1}, where $\beta^* = \beta^{\frac{1-\alpha}{1-\rho}}$, $\eta = \frac{\rho-\alpha}{1-\rho}$, $\theta = -\rho\frac{1-\alpha}{1-\rho}$, and R_{t+1}^m is the (gross) return of the optimal portfolio (R_{t+1}^m is the return from period t to t+1 of

GMM package.

a security that pays c_t every period forever). They use the value-weighted return of shares traded on the New York Stock Exchange as R^m_{t+1}. Thus, the Roll's (1977) critique of CAPM is relevant here as discussed.

Even though (10.11) holds for $R_{t+1} = R^m_{t+1}$, the identification assumption discussed in Section 9.3 is violated for this choice of R_{t+1} because there exists a trivial solution,$(\beta^*, \eta, \theta) = (1, -1, 0)$, for $g(\mathbf{x}_t, \mathbf{b}) = \mathbf{0}$. When multiple returns that include R^m_{t+1} are used simultaneously, then the whole system can satisfy the identification assumption, but the GMM estimators for this partially unidentified system are likely to have bad small sample properties. A similar problem arises when R_{t+1} does not include R^m_{t+1} but includes multiple equity returns whose linear combination is close to R^m_{t+1}. It should be noted that Epstein and Zin avoid these problems by carefully choosing returns to be included as R_{t+1} in their system.

10.4 Time Aggregation

The use of consumption data for C-CAPM is subject to a time aggregation problem (see, e.g., Hansen and Sargent, 1983a,b) because consumers can make decisions at intervals much finer than the observed frequency of the data and because the observed data consist of average consumption over a period of time.

In linear models for which the disturbance before time aggregation is a martingale difference, time aggregation means that the disturbance has an MA(1) structure and the instrumental variables need to be lagged an additional period. See, e.g, Grossman, Melino, and Shiller (1987), Hall (1988), and Hansen and Singleton (1996) for applications to C-CAPM and Heaton (1993) and Christiano, Eichenbaum, and Marshall (1991) for applications to Hall (1978) type permanent income models.

In nonlinear models for which the disturbance before time aggregation is a martingale difference, time aggregation has more complicated effects. Allowing the disturbance to have an MA(1) structure and letting instrumental variables lagged an additional period do not completely eliminate the effects caused by time aggregation. Nevertheless, these methods are often used to mitigate time aggregation problems in applications (see, e.g., Epstein and Zin, 1991; Ogaki and Reinhart, 1998b).

For nonlinear models, one way to use GMM to take into account the full effects of time aggregation is to combine GMM with simulations. For example, Heaton (1995) uses the method of simulated moments (MSM) for his nonlinear asset pricing model with time-nonseparable preferences in taking time aggregation into account. Bossaerts (1988), Duffie and Singleton (1993), McFadden (1989), Pakes and Pollard (1989), Lee and Ingram (1991), and Pearson (1991), among others, have studied asymptotic properties of MSM.

10.5 Multiple-Goods Models

Mankiw, Rotemberg, and Summers (1985), Dunn and Singleton (1986), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1990), and Osano and Inoue (1991), among others, have estimated versions of multiple-good C-CAPM. Basic economic formulations of these multiple-good models will be illustrated in the context of a simple model with one durable good and one nondurable good.

Let us replace (9.2) by Houthakker's (1960) addilog utility function that Miron (1986), Ogaki (1988, 1989), and Osano and Inoue (1991) among others have estimated:

$$U(c_t, d_t) = \frac{1}{1-\alpha}(c_t^{1-\alpha} - 1) + \frac{\theta}{1-\eta}(k_t^{1-\eta} - 1), \tag{10.12}$$

where c_t is nondurable consumption and k_t is household capital stock from purchases of durable consumption good d_t.[4] The stock of durables is assumed to depreciate at a constant rate $1-a$, where $0 \leq a < 1$:

$$k_t = ak_{t-1} + d_t. \tag{10.13}$$

Alternatively, k_t can be considered as a service flow in (10.8) with $a_\tau = a^\tau$. When $\alpha \neq \eta$, preferences are not quasi homothetic. In practice, the data for k_t is constructed from data for an initial stock k_0, and for d_t for $t = 1, \cdots, T$. Let p_t be the intratemporal relative price of durable and nondurable consumption. Then the intraperiod first order condition that equates the relative price with the marginal rate of substitution is

$$p_t = \frac{\theta E(\sum_{\tau=1}^{\infty} \beta^\tau a^\tau k_{t+\tau}^{-\eta} | \mathrm{I}_t)}{c_t^{-\alpha}}. \tag{10.14}$$

Assume that $\frac{d_{t+1}}{d_t}$ is stationary. Then $\frac{k_{t+\tau}}{d_t}$ is stationary for any τ because $\frac{k_{t+\tau}}{d_t} = \sum_{i=0}^{\infty} a^i \frac{d_{t+\tau-i}}{d_t}$. From (10.14),

$$\frac{p_t c_t^{-\alpha}}{d_t^{-\eta}} = \theta E[\sum_{\tau=1}^{\infty} \beta^\tau a^\tau (\frac{k_{t+\tau}}{d_t})^{-\eta} | \mathrm{I}_t]. \tag{10.15}$$

Assume that the variables in I_t are stationary.[5] Then (10.15) implies that the $p_t \frac{c_t^{-\alpha}}{d_t^{-\eta}}$ is stationary because the right hand side of (10.15) is stationary. Taking natural logs, we conclude that $\ln(p_t) - \alpha \ln(c_t) + \eta \ln(d_t)$ is stationary. This restriction is called the stationarity restriction.

From (10.14), define

$$\epsilon_t^0 = p_t c_t^{-\alpha} - (1 - \beta a F)^{-1} \theta k_t^{-\eta}, \tag{10.16}$$

[4] Since the addilog utility function is not quasi-homothetic in general, the distribution of initial wealth affects the utility function of the representative consumer. The existence of a representative consumer under complete markets is discussed by Ogaki (1990) for general concave utility functions and by Atkeson and Ogaki (1996) for extended addilog utility functions.

[5] If I_t includes nonstationary variables, assume that the right hand side of (10.14) is the same as the expectation conditioned on the stationary variables in I_t.

where F is the forward operator. The first order condition (10.14) implies that $E(\epsilon_t^0|\mathrm{I}_t) = 0$. One problem is that ϵ_t^0 involves $k_{t+\tau}$ for τ from 0 to infinity, so that ϵ_t^0 cannot be used as the disturbance for GMM. To solve this problem, define $\epsilon_t = (1 - \beta a F)\epsilon_t^0$. Note that ϵ_t involves only $c_t, c_{t+1}, p_t, p_{t+1}$, and k_t and that $E[\epsilon_t|\mathrm{I}_t] = 0$. Hence ϵ_t forms the basis of GMM. The only remaining problem is attaining stationarity. One might think it is enough to divide ϵ_t by $k_t^{-\eta}$, so that the resulting ϵ_t is stationary as implied by the stationarity restriction. It should be noted that it is *not* enough for $\epsilon_t = g(\mathbf{x}_t, \mathbf{b}_0)$ to be stationary, rather it is also necessary for $g(\mathbf{x}_t, \mathbf{b})$ to be stationary for $\mathbf{b} \neq \mathbf{b}_0$. Hence if α and η are unknown and c_t or d_t is difference stationary, GMM cannot be applied to the first order condition (10.14).[6] Ogaki (1988, 1989) assumes that c_t and d_t are trend stationary and applies the method of Section 10.2 above to utilize the detrended version of ϵ_t. In these applications, the restrictions on the trend coefficients and the curvature parameters α and η implied by the stationarity restriction are imposed on the GMM estimators. Imposing the stationarity restrictions also lead to more reasonable point estimates for α and η.

Eichenbaum, Hansen, and Singleton (1988) and Eichenbaum and Hansen (1990) use the Cobb-Douglas utility function, so that α and η are known to be one.[7] They allow preferences to be nonseparable across goods and time-nonseparable, but the stationarity restriction is shown to hold. In this case, the stationarity restriction implies that $p_t \frac{c_t^{-1}}{k_t^{-1}}$ is stationary. This transformation does not involve any unknown parameters. Hence, this transformation is used to apply GMM to their intraperiod first order conditions.

[6]Cointegrating regressions can be used for this case as explained below.

[7]Also see Ogaki (1988) for a discussion of the stationarity restriction implied by the Cobb-Douglas utility function.

10.6 Seasonality

Miron (1986) augments the Hansen and Singleton's (1982) model by including deterministic seasonal taste shifters and argues that the empirical rejection of C-CAPM by Hansen and Singleton (1982) and others might be attributable to the use of seasonally adjusted data.[8] Although this is theoretically possible, English, Miron, and Wilcox (1989) find that seasonally unadjusted quarterly data reject asset pricing equations at least as strongly as seasonally adjusted data.[9] Ogaki (1988) also finds similar empirical results for seasonally unadjusted and adjusted data in the system that involves both asset pricing equations and intraperiod first order conditions.

Singleton (1988) argues that the inclusion of taste shifters in C-CAPM is essentially equivalent to directly studying consumption data with deterministic seasonality removed. This finding results because we do not obtain much identifying information from seasonal fluctuations about preferences if most of the seasonal fluctuations come from seasonal taste shifts.[10] On the other hand, seasonal fluctuations may contain useful identifying information about the production functions if production functions are relatively stable over the seasonal cycle. Braun and Evans (1998) utilize such identifying information.

Masao needs to check this!

Ferson and Harvey (1992) construct seasonally unadjusted monthly data and estimate a C-CAPM with time nonseparable preferences. They find that seasonal habit persistence is empirically significant. Heaton (1993) also finds evidence for

[8]It should be noted that a deterministic seasonal dummy can be viewed as an artificial stationary and ergodic stochastic process (see, e.g., Ogaki, 1988, pp. 26–27). Hence, GMM can be applied to models with deterministic seasonal taste shifts.

[9]Hoffman and Pagan (1989) also obtain similar results.

[10]Beaulieu and Miron (1991) cast doubt on the view that negative output growth in the first quarter (see, e.g., Barsky and Miron, 1989) is caused by negative technology seasonal by observing negative output growth in the Southern Hemisphere.

seasonal habit formation in Hall (1978) type permanent income models.[11]

10.7 Monetary Models

In some applications, monetary models are estimated by applying GMM to Euler equations and/or intratemporal first order conditions. Singleton (1985), Ogaki (1988), Finn, Hoffman, and Schlagenhauf (1990), Bohn (1991), and Sil (1992) estimate cash-in-advance models, Poterba and Rotemberg (1987), Eckstein and Leiderman (1989), and Finn, Hoffman, and Schlagenhauf (1990), Imrohoroglu (1991) estimate money-in-the-utility-function (MIUF) models, and Marshall (1992) estimates a transactions-cost monetary model.

Cash-in-advance models involve only minor variations on the asset pricing equation (10.1) as long as the cash-in-advance constraints are binding and c_t is a cash good (in the terminology of Lucas and Stokey, 1987). However, nominal prices of consumption, nominal consumption, and nominal asset returns are aligned over time in a different way in monetary models than they are in the Hansen and Singleton's (1982) model. Information available to agents at time t is also considered in a different way. As a result, instrumental variables are lagged one period more than in the Hansen-Singleton model, and $\mathbf{u}_t$ has an MA(1) structure (time aggregation has the same effects in linear models as discussed above). There is some tendency for the chi-square test statistics for the overidentifying restrictions to be more favorable for the timing conventions suggested by cash-in-advance models (see Finn, Hoffman, and Schlagenhauf, 1990; Ogaki, 1988). Ogaki (1988) focuses on monetary distortions in relative prices for a cash good and a credit good and does not find monetary

[11] See Ghysels (1990, especially Section I.3) for a survey of the economic and econometric issues of seasonality.

distortions in the U.S. data he examines.

10.8 Calculating Standard Errors for Estimates of Standard Deviation, Correlation, and Autocorrelation

In many macroeconomic applications, researchers report estimates of standard deviations, correlations, and autocorrelations of economic variables. It is possible to use a GMM program to calculate standard errors for these estimates, in which the serial correlation of the economic variables is taken into account (see, e.g., Backus, Gregory, and Zin, 1989; Backus and Kehoe, 1992).

For example, let x_t and y_t be economic variables of interest that are assumed to be stationary. Let $\mathbf{x}_t = (x_t, y_t)$ and $f(\mathbf{x}_t, \mathbf{b}) = (x_t, x_t^2, y_t, y_t^2, x_t y_t, x_t x_{t-1})' - \mathbf{b}$, where $f(\mathbf{x}_t, \mathbf{b})$ is a disturbance defined at time t and a quadratic form of its sample average is the objective function to be minimized in GMM estimation. Then the parameters to be estimated are the population moments; $\mathbf{b}_0 = (E(x_t), E(x_t^2), E(y_t), E(y_t^2), E(x_t y_t), E(x_t x_{t-1}))$. Applying GMM to $f(\mathbf{x}_t, \mathbf{b})$, one can obtain an estimate of $\mathbf{b}_0$, $\mathbf{b}_T$, and an estimate of covariance matrix of $T^{\frac{1}{2}}(\mathbf{b}_T - \mathbf{b}_0)$.[12] In most applications, the order of serial correlation of $(x_t, x_t^2, y_t, y_t^2, x_t y_t, x_t x_{t-1})'$ is unknown, and its long-run covariance matrix, $\mathbf{\Omega}$, can be estimated by any method in Chapter 6 (such as Andrews and Monahan's prewhitened QS kernel estimation method).

Standard deviations, correlations, and autocorrelations are nonlinear functions of $\mathbf{b}_0$. Hence, one can use the delta method to calculate the standard errors of the

[12] The covariance matrix $Cov(\mathbf{\Omega}^{-1})$ is defined in (9.13). In this particular example, $Cov(\mathbf{\Omega}^{-1})$ coincides with the long-run variance of $f(\mathbf{x}_t, \mathbf{b}_0)$ because the derivative of $f(\mathbf{x}_t, \mathbf{b}_0)$ is an identity. More generally, if more moment conditions are added to make the system overidentified, then $Cov(\mathbf{\Omega}^{-1})$ will be different from the long-run covariance matrix.

estimates of these statistics. Let $a(\mathbf{b}_0)$ be the statistic of interest. Continuing the example above, imagine that a researcher is estimating the standard deviation of x_t. Then $a(\mathbf{b}_0) = \sqrt{\mathrm{var}(x_t)} = (E(x_t^2) - E(x_t)^2)^{\frac{1}{2}} = (b_{02} - b_{01}^2)^{\frac{1}{2}}$, where $b_{01} = E(x_t)$, $b_{02} = E(x_t^2)$ and $a(\mathbf{b}_T)$ is a consistent estimator of $a(\mathbf{b}_0)$. If we apply the delta method explained in Proposition 5.8, $\sqrt{T}(a(\mathbf{b}_T) - a(\mathbf{b}_0))$ has an approximate normal distribution with the variance $\mathbf{d}(\mathbf{b}_0)Cov(\mathbf{\Omega}^{-1})\mathbf{d}(\mathbf{b}_0)'$ in large samples, where $\mathbf{d}(\mathbf{b}_0)$ is the derivative of $a(\cdot)$ evaluated at $\mathbf{b}_0$.

There is a pitfall that should be avoided in setting the GMM momment conditions in these applications. The parameters can enter the GMM moment conditions in nonlinear ways, but the sample moments should not. For example, it may be tempting to estiamte the variance of x_t in the above example by setting the moment condition to be $b - (x_t - \bar{x})^2$ where b is the variance to be estiamted and *overvarx* is the sample mean. However, because the sample mean enters the GMM moment condition in a nonlinear way, $E(b - (x_t - \bar{x})^2)$ is not equal to zero. This pitfall can be easily avoided by estimating $E(x)$ and $E(x^2)$ as in the example above.

An example of a problematic application with this type of the pitfall can be found in Section 5 of Ambler, Cardia, and Zimmermann (2004). In estimating a pair of correlations, their estimate is a solution to the problem of minimizing

$$\{\frac{1}{T}\sum_{t=1}^{T}(\boldsymbol{\rho} - \bar{\boldsymbol{\rho}}_t)\}'\mathbf{W}_T\{\frac{1}{T}\sum_{t=1}^{T}(\boldsymbol{\rho} - \bar{\boldsymbol{\rho}}_t)\} \tag{10.17}$$

where the parameter $\boldsymbol{\rho}$ is a $2x1$ vector of the population correlations of four varaibles (say x_{it} for $i = 1, 2, 3, 4$), and $\bar{\boldsymbol{\rho}}_t$ is a $2x1$ vector whose first element is given by

$$\bar{\rho_{1t}} = \frac{(x_1 - \bar{x_1})(x_2 - \bar{x_2})}{\bar{\sigma}_1\bar{\sigma}_2} \tag{10.18}$$

and whose second element is given by

$$\bar{\rho_{2t}} = \frac{(x_3 - \bar{x_3})(x_4 - \bar{x_4})}{\bar{\sigma}_3\bar{\sigma}_4}. \tag{10.19}$$

Here $\bar{x}_i$ is the sammple mean and $\bar{\sigma}_i$ is the sample variance of x_i. This set-up resembles that of GMM, but cannot be embedded in the standard GMM framework. This is because the sample mean dna the sample variance enter the moment conditions in nonlinear ways.

10.9 Dynamic Stochastic General Equilibrium Models and GMM Estimation

Real Business Cycle Models and other Dynamic Stochastic General Equilibrium (DSGE) models can be estimated and tested by GMM. These models are often simulated and the results are evaluated without considering sampling errors. GMM gives a simple method to take into account sampling errors. Such a method was originally developed by Christiano and Eichenbaum (1992). A survey by Burnside (1999) describes how GMM estimation is used for real business cycle models and explains how to use the programs written by the author. Recent applications of GMM to DSGE models include Alexopoulos (2004) and Aguiar and Gopinath (2007). In this section, we explain a method used by Burnside, Eichenbaum, and Rebelo (1993) using a simpler model than these authors used. This method uses results in King, Plosser, and

Rebelo (1988a,b) that show how the model parameters are related to the moments of economic variables.

Consider a social planner's problem:

$$\max_{C_t, K_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{10.20}$$
$$\begin{aligned} \text{s.t} \quad Y_t &= A_t K_{t-1}^{\alpha} = C_t + I_t \\ I_t &= K_t - (1-\delta)K_{t-1} \\ \ln A_t &= \rho \ln A_{t-1} + \epsilon_t, \end{aligned}$$

where C_t is consumption, K_t is a capital stock, Y_t is output, I_t is investment, $1-\delta$ is a depreciation rate, and A_t represents the level of technology. We do not include labor to simplify the model for the pedagogical purpose. Using the budget constraint given by

$$C_t = A_t K_{t-1}^{\alpha} - K_t + (1-\delta)K_{t-1} \tag{10.21}$$

the first order condition becomes

$$-U'(C_t) + \beta E_t U'(C_{t+1})(\alpha A_{t+1} K_t^{\alpha-1} + 1 - \delta) = 0. \tag{10.22}$$

In a steady state, we have $\epsilon_t = 0$ so that $A_t = 1$. We can also take out the expectation as C_t, K_t, and Y_t are constants. Thus, (10.22) implies

$$\beta(\alpha K^{\alpha-1} + 1 - \delta) = 1. \tag{10.23}$$

From (10.23) we can calculate the steady state solutions

$$\begin{aligned} K^* &= \left(\frac{1}{\alpha}\left(\frac{1}{\beta} - (1-\delta)\right)\right)^{\frac{1}{\alpha-1}} \\ C^* &= {K^*}^{\alpha} - \delta K^* \\ Y^* &= {K^*}^{\alpha}. \end{aligned} \tag{10.24}$$

For solutions in other states, we need to take a log linearization using $y = \ln x$ and

$$
\begin{aligned}
f(x) &= f(e^y) \\
&= f(e^{y_0}) + \frac{\partial f(e^{y_0})}{\partial y}(y - y_0) \\
&= f(x_0) + \frac{\partial f(x_0)}{\partial x}\frac{1}{\partial \ln x_0/\partial x}(\ln x - \ln x_0) \\
&= f(x_0) + f'(x_0)x_0(\ln x - \ln x_0)
\end{aligned} \tag{10.25}
$$

Plug (10.25) into (10.22), then

$$
\begin{aligned}
U'(C_0) + U''(C_0)C_0\hat{C}_t &= \beta U'(C_0)(\alpha A_0 K_0^{\alpha-1} + 1 - \delta) \\
&+ \beta E_t U''(C_0)C_0(\alpha A_0 K_0^{\alpha-1} + 1 - \delta)\hat{C}_{t+1} \\
&+ \beta E_t U'(C_0)\alpha A_0 K_0^{\alpha-1}\hat{A}_{t+1} \\
&+ \beta E_t U''(C_0)\alpha(\alpha - 1)A_0 K_0^{\alpha-1}\hat{K}_t
\end{aligned} \tag{10.26}
$$

where $\hat{C}_t = \ln C_t - \ln C_0$, $\hat{A}_t = \ln A_t - \ln A_0$, and $\hat{K}_t = \ln K_t - \ln K_0$. By the property of the steady state, constant terms are cancelled out so that

$$
\begin{aligned}
& U''(C_0)C_0\hat{C}_t = \beta E_t U''(C_0)C_0(\alpha A_0 K_0^{\alpha-1} + 1 - \delta)\hat{C}_{t+1} \\
+ \;& \beta E_t U'(C_0)\alpha A_0 K_0^{\alpha-1}\hat{A}_{t+1} + \beta E_t U''(C_0)\alpha(\alpha - 1)A_0 K_0^{\alpha-1}\hat{K}_t,
\end{aligned} \tag{10.27}
$$

where $E_t\hat{A}_{t+1} = \rho\hat{A}_t$. Thus, this equation can be simplified by

$$
\hat{C}_t = \tilde{A}_c E_t \hat{C}_{t+1} + \tilde{A}_k \hat{K}_t + \tilde{A}_a \hat{A}_t. \tag{10.28}
$$

Since this equation contains two control variables, we further simplify it by replacing $\hat{C}_t$ with the following log linearization of (10.21):

$$
\hat{C}_t = A_0 K_0^{\alpha}\hat{A}_t + \alpha A_0 K_0^{\alpha}\hat{K}_{t-1} - K_0\hat{K}_t + (1 - \delta)K_0\hat{K}_{t-1}. \tag{10.29}
$$

and finally we get

$$E_t\hat{K}_{t+1} + A_1\hat{K}_t + A_2\hat{K}_{t-1} = A_3\hat{A}_t. \tag{10.30}$$

Let $L^{-1}x_t$ denote $E_t x_{t+1}$, then (10.30) can be expressed by

$$(1 - B_1L^{-1})(1 - B_2L)\hat{K}_t = B_3\hat{A}_t \tag{10.31}$$

or

$$\begin{aligned}(1 - B_2L)\hat{K}_t &= (1 - B_1L^{-1})B_3\hat{A}_t \\ &= B_3\sum_{i=0}^{\infty} E_tB_1^i\hat{A}_{t+i} \\ &= B_3\sum_{i=0}^{\infty} B_1^i\rho^i\hat{A}_t \\ &= B_3\sum_{i=0}^{\infty} B_1^i\rho^i\hat{A}_t.\end{aligned} \tag{10.32}$$

Thus, the solution of the model is given by

$$\hat{K}_t = C_{11}\hat{K}_{t-1} + C_{12}\hat{A}_t. \tag{10.33}$$

We can also get the solution for C_t by plugging (10.33) into (10.29):

$$\hat{C}_t = C_{21}\hat{K}_{t-1} + C_{22}\hat{A}_t. \tag{10.34}$$

In general, we can always express the solutions of the model by

$$\begin{aligned}x_{t+1} &= \gamma_{xx}x_t + \gamma_{xz}z_t \\ \lambda_t &= \gamma_{\lambda x}x_t + \gamma_{\lambda z}z_t \\ u_t &= \gamma_{ux}x_t + \gamma_{uz}z_t,\end{aligned} \tag{10.35}$$

where x_t is a vector of state variables (K_{t-1}), λ_t is a costate variable, u_t is a vector of control variables (C_t), and z_t is a vector of exogenous variables (A_t). Let the law of motion for the exogenous variables be

$$z_t = \pi z_{t-1} + \epsilon_t, \tag{10.36}$$

then we get

$$\begin{aligned} \begin{bmatrix} x_{t+1} \\ z_{t+1} \end{bmatrix} &= \begin{bmatrix} \gamma_{xx} & \gamma_{xz} \\ 0 & \pi \end{bmatrix} \begin{bmatrix} x_t \\ z_t \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{t+1} \end{bmatrix} \\ &= M s_t + \hat{\epsilon}_{t+1} \end{aligned} \tag{10.37}$$

or

$$s_{t+1} = M s_t + \hat{\epsilon}_{t+1}, \tag{10.38}$$

where $s_t = (x_t, z_t)'$. Let f_t be other variables of interest characterized by $f_t = Fcu_t + F_x x_t + F_z z_t$, then

$$\begin{aligned} \begin{bmatrix} \lambda_t \\ u_t \\ f_t \end{bmatrix} &= \begin{bmatrix} \gamma_{\lambda x} & \gamma_{\lambda z} \\ \gamma_{ux} & \gamma_{uz} \\ F_c\gamma_{ux} + F_x & F_c\gamma_{uz} + F_z \end{bmatrix} s_t \\ &= H s_t. \end{aligned} \tag{10.39}$$

Therefore, provided with the parameters in the first order conditions and those in the law of motion for the exogenous variables, we can compute M and H. GMM is used to estimate the parameters. Once M and H are derived, we can compute the impulse response function and the autocovariance implied by the model. By taking an MA representation of (10.38), the h-step impulse response function of the $i-th$ variable of $(\lambda_t, u_t, f_t)'$ on the $j-th$ shock of $\hat{\epsilon}_t$ is given by

$$(HM^h)_{(i,j)}. \tag{10.40}$$

The autocovariance is computed by

$$\begin{aligned} \Gamma_i &= E(s_t s'_{t-i}) \\ &= M^i \Gamma_0, \end{aligned} \tag{10.41}$$

where $\Gamma_0 = E(s_t s_t')$ that is computed as follows. Let $M = VDV^{-1}$ where D is a diagonal matrix that consists of eigen-values of M, and V is a matrix of corresponding eigen-vectors. By pre-multiplying V^{-1} on the both sides of (10.38), we get

$$V^{-1}s_{t+1} = DV^{-1}s_t + V^{-1}\hat{\epsilon}_{t+1} \tag{10.42}$$

or

$$\tilde{s}_{t+1} = D\tilde{s}_t + \tilde{\epsilon}_{t+1}. \tag{10.43}$$

Thus, we can compute the transformed autocovariance by

$$\begin{aligned} \tilde{\Gamma}_{0,ij} &= E(s_{it}s_{jt}') \\ &= \frac{1}{1-d_i d_j}\tilde{\Sigma}_{i,j} \end{aligned} \tag{10.44}$$

and

$$\Gamma_0 = V\tilde{\Gamma}_0 V'. \tag{10.45}$$

We can also compute the autocovariance of other variables using

$$\begin{aligned} E(s_t w_{t-i}') &= E(s_t(Hs_{t-i})') = M^i\Gamma_0 H' \\ E(w_t w_{t-i}') &= E(Hs_t(Hs_{t-i})') = HM^i\Gamma_0 H'. \end{aligned} \tag{10.46}$$

10.10 GMM and an ARCH Process

As explained in Chapter 2, an autoregressive conditional heteroskedastic (ARCH) process is frequently employed to model conditional heteroskedasticity. A typical estimation method for an ARCH model is the Maximum Likelihood (ML) estimator with the assumption that the conditional distribution of the error term follows normal or t-distribution (see Bollerslev, Chou, and Kroner, 1992, for survey). However, ARCH models can also be estimated by GMM, which produces consistent estimates of the parameters without a specific distributional assumption (see, e.g., Mark, 1988; Simon, 1989). Further, as Rich, Raymond, and Butler (1991) point out, the GMM estimation directly allows for the specification test introduced by Hansen (1982).

An ARCH process is modeled as an innovation in the mean for some other stochastic process in most applications. Consider a regression model with ARCH(q) disturbances.

$$y_t = \mathbf{x}_{2,t}'\boldsymbol{\beta} + \epsilon_t \tag{10.47}$$

$$E(\epsilon_t \mid \mathrm{I}_{t-1}) = 0 \tag{10.48}$$

$$E(\epsilon_t^2 \mid \mathrm{I}_{t-1}) = h_t \tag{10.49}$$

$$h_t = \alpha + \sum_{i=1}^{q} \gamma_i \epsilon_{t-i}^2; \quad \alpha > 0, \quad \sum_{i=1}^{q} \gamma_i < 1, \quad \gamma_i \geq 0 \tag{10.50}$$

where y_t is the dependent variable, $\mathbf{x}_{2,t}$ is a vector of explanatory variables in the information set I_{t-1} which is assumed to be $\mathrm{I}_{t-1} \subset \mathrm{I}_t$ for any t and $\boldsymbol{\beta}, \alpha$ and γ are fixed parameters.

To apply GMM, Rich, Raymond, and Butler (1991) rewrite equations (10.47) and (10.49) as:

$$y_t = \mathbf{x}_{2,t}'\boldsymbol{\beta} + \epsilon_t \tag{10.51}$$

$$\epsilon_t^2 = \alpha + \sum_{i=1}^{q} \gamma_i \epsilon_{t-i}^2 + \eta_t \tag{10.52}$$

where

$$\eta_t = \epsilon_t^2 - h_t, \quad E(\eta_t \mid \mathrm{I}_{t-1}) = 0 \tag{10.53}$$

From these, we can obtain a system of two equations describing the innovations to the mean and variance of the ARCH(q) process, respectively,

$$\epsilon_t = y_t - \mathbf{x}_{2,t}'\boldsymbol{\beta} \tag{10.54}$$

$$\eta_t = (y_t - \mathbf{x}_{2,t}'\boldsymbol{\beta})^2 - \alpha - \sum_{i=1}^{q} \gamma_i (y_{t-i} - \mathbf{x}_{2,t-i}'\boldsymbol{\beta})^2 \tag{10.55}$$

Let $\tilde{\mathbf{b}}$ be the n-dimensional vector of parameters $(\tilde{\boldsymbol{\beta}}', \tilde{\alpha}, \tilde{\boldsymbol{\gamma}}')'$ of the ARCH model and $\mathbf{x}_t = (y_t, \mathbf{x}_{2,t}')'$. Let $\mathbf{g}(\mathbf{x}_t, \tilde{\mathbf{b}})$ be a 2-dimensional vector of functions, then

$$\mathbf{g}(\mathbf{x}_t, \mathbf{b}_0) = \begin{bmatrix} \epsilon_t(\boldsymbol{\beta}) \\ \eta_t(\boldsymbol{\beta}, \alpha, \boldsymbol{\gamma}) \end{bmatrix} \tag{10.56}$$

$$E(\mathbf{g}(\mathbf{x}_t, \mathbf{b}_0) \mid \mathrm{I}_{t-1}) = \mathbf{0} \tag{10.57}$$

where $\mathbf{b}_0 = (\boldsymbol{\beta}', \alpha, \boldsymbol{\gamma}')'$ is the true parameter.

Suppose $\mathbf{z}_{t-1}^1$ and $\mathbf{z}_{t-1}^2$ are an (m_1 x 1) and an (m_2 x 1) vector of random variables in the information set I_{t-1}, uncorrelated with ϵ_t and η_t, respectively, to serve as instrumental variables. Let $\mathbf{z}_{t-1}$ be (m x 2) block diagonal matrix where $m = m_1 + m_2$,

$$\mathbf{z}_{t-1} = \begin{bmatrix} \mathbf{z}_{t-1}^1 & 0 \\ 0 & \mathbf{z}_{t-1}^2 \end{bmatrix} \tag{10.58}$$

By the law of iterative expectations, we obtain unconditional moment restrictions:

$$E(\mathbf{z}_{t-1}\, \mathbf{g}(\mathbf{x}_t, \mathbf{b}_0)) = \mathbf{0} \tag{10.59}$$

Equation (10.59) represents a set of m orthogonality conditions which are used to estimate $\mathbf{b}_0$ with $\mathbf{z}_{t-1}$ serving instruments in the ARCH model. Based on this procedure, Rich, Raymond, and Butler (1991) obtain results similar to ML estimates of Engle and Kraft's (1983) ARCH model of U.S. inflation.

This GMM framework can be extended to the generalized ARCH model, GARCH(p,q), where equation (10.50) allows for autoregressive components in the heteroskedastic variance:

$$h_t = \alpha + \sum_{i=1}^{q} \gamma_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \delta_j h_{t-j} \tag{10.60}$$

where $\alpha > 0$, $\sum_{i=1}^{q} \gamma_i < 1, \gamma_i \geq 0$, $\sum_{j=1}^{p} \delta_j < 1, \delta_j \geq 0$. In this case, we can still get the same moment conditions, equation (10.59), where $\mathbf{b}_0 = (\boldsymbol{\beta}', \alpha, \boldsymbol{\gamma}', \boldsymbol{\delta}')'$ is the true parameter.

10.11 Estimation and Testing of Linear Rational Expectations Models

In this section, econometric methods that impose and test the restrictions implied by linear rational expectations models are described. Many linear rational expectations models imply that an economic variable depends on a geometrically declining weighted sum of expected future values of another variable

$$y_t = aE(\sum_{i=1}^{\infty} \beta^i x_{t+i} | \mathrm{I}_t) + \mathbf{c}'\mathbf{z}_t, \tag{10.61}$$

where a and β are constants, $\mathbf{c}$ is a vector of constants, y_t and x_t are random variables, and $\mathbf{z}_t$ is a random vector. This implication imposes nonlinear restrictions on the VAR representation of y_t, x_t, and $\mathbf{z}_t$ as shown by Hansen and Sargent (1980). In

Section 10.11.1, these nonlinear restrictions are discussed. Section 10.11.2 describes econometric methods to utilize these restrictions.

10.11.1 The Nonlinear Restrictions

Consider West's (1987) model as an example of linear rational expectations model. Let p_t be the real stock price (after the dividend is paid) in period t and d_t be the real dividend paid to the owner of the stock at the beginning of period t. Then the arbitrage condition is

$$p_t = E[\beta(p_{t+1} + d_{t+1})|\mathrm{I}_t], \tag{10.62}$$

where β is the constant real discount rate, I_t is the information set available to economic agents in period t. Solving (10.62) forward and imposing the no bubble condition, we obtain the present value formula:

$$p_t = E(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{I}_t). \tag{10.63}$$

We now derive restrictions for p_t and d_t implied by (10.63). Many linear rational expectations models imply that a variable is the expectation of a discounted infinite sum conditional on an information set. Hence similar restrictions can be derived for these rational expectations models. We consider two cases, depending on whether d_t is assumed to be covariance stationary or is unit root nonstationary.

Assume that d_t is covariance stationary with mean zero (imagine that data are demeaned), so that it has a Wold moving average representation

$$d_t = \alpha(L)\nu_t, \tag{10.64}$$

where $\alpha(L) = 1 + \alpha_1 L + \alpha_2 L^2 + \cdots$ and where

$$\nu_t = d_t - \hat{E}(d_t|\mathrm{H}_{t-1}). \tag{10.65}$$

Here, $\hat{E}(\cdot|\mathrm{H}_t)$ is the linear projection operator onto the information set $\mathrm{H}_t = \{d_t, d_{t-1}, d_{t-2}, \cdots\}$. We assume that the econometrician uses the information set H_t, which may be much smaller than the economic agents' information set, I_t. Assuming that $\alpha(L)$ is invertible,

$$\phi(L)d_t = \nu_t, \tag{10.66}$$

where $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots$.

Using (10.63) and the law of iterated projections, we obtain

$$p_t = \hat{E}(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{H}_t) + w_t, \tag{10.67}$$

where

$$w_t = E(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{I}_t) - \hat{E}(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{H}_t), \tag{10.68}$$

and $\hat{E}(w_t|\mathrm{H}_t) = 0$. Since $\hat{E}(\cdot|\mathrm{H}_t)$ is the linear projection operator onto H_t,

$$\hat{E}(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{H}_t) = \delta(L)d_t, \tag{10.69}$$

where $\delta(L) = \delta_1 + \delta_2 L + \cdots$. Following Hansen and Sargent (1980, Appendix A), we obtain the restrictions imposed by (10.69) on $\delta(L)$ and $\phi(L)$. The left-hand side of (10.69) can be written

$$\begin{aligned}
\hat{E}(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{H}_t) &= \hat{E}\left(\frac{\beta L^{-1}}{1-\beta L^{-1}} d_t|\mathrm{H_t}\right) \\
&= \left[\frac{\beta L^{-1}\alpha(L)}{1-\beta L^{-1}}\right]_+ \nu_t
\end{aligned} \tag{10.70}$$

where $[B(L)]_+$ is an annihilator that removes negative power of the lag polynomial $B(L)$. The second equality holds because ν_t is fundamental. Then by replacing L with z in (10.70), we have

$$\frac{\beta z^{-1}\alpha(z)}{1-\beta z^{-1}} = \frac{\beta z^{-1}(\alpha(z) - \alpha(\beta))}{1-\beta z^{-1}} + \frac{\beta z^{-1}\alpha(\beta)}{1-\beta z^{-1}}. \tag{10.71}$$

Note that the first term in the right-hand side is removable singularity and the second term has only negative power of lag polynomial that is to be removed by the annihilator. Therefore we can write (10.70) as

$$
\begin{aligned}
(10.72) \qquad \left[\frac{\beta L^{-1}\alpha(L)}{1-\beta L^{-1}}\right]_+ \nu_t &= \left[\frac{\beta z^{-1}(\alpha(z)-\alpha(\beta))}{1-\beta L^{-1}} + \frac{\beta L^{-1}\alpha(\beta)}{1-\beta L^{-1}}\right]_+ \nu_t \\
&= \frac{\beta L^{-1}(\alpha(L)-\alpha(\beta))}{1-\beta L^{-1}}\nu_t.
\end{aligned}
$$

Since $\nu_t = \phi(L)d_t$ as in (10.64), we have the following restriction

$$
\begin{aligned}
(10.73) \qquad \delta(L) &= \frac{\beta L^{-1}(\alpha(L)-\alpha(\beta))}{1-\beta L^{-1}}\,\phi(L) \\
(10.74) \qquad &= \frac{\beta L^{-1}(1-\phi^{-1}(\beta)\phi(L))}{1-\beta L^{-1}}.
\end{aligned}
$$

We now parameterize $\phi(L)$ as a q-th order polynomial:

$$
(10.75) \qquad d_t = \phi_1 d_{t-1} + \cdots + \phi_q d_{t-q} + \nu_t.
$$

Then, by using state space representation, (10.75) can be written as

$$
(10.76) \qquad D_t = AD_{t-1} + V_t
$$

where $D_t = (d_t, d_{t-1}, \ldots, d_{t-q+1})'$ and

$$
(10.77) \qquad A = \begin{bmatrix} \phi_1 & \cdots & \cdots & \phi_q \\ 1 & & & 0 \\ & \ddots & & \vdots \\ & & 1 & 0 \end{bmatrix}
$$

Then (10.67) can be written as

$$
\begin{aligned}
(10.78) \qquad p_t &= \hat{E}\left(\sum_{i=1}^{\infty} \beta^i d_{t+i} | \mathrm{H}_t\right) + w_t \\
&= e_1 \beta A (I-\beta A)^{-1} D_t + w_t
\end{aligned}
$$

where $e_1 = (1, 0, \ldots, 0)'$.

Also (10.73) is used to show that $\delta(L)$ is a finite order polynomial and to give a explicit formula for the coefficients for $\delta(L)$.[13] Thus

$$p_t = \delta_1 d_t + \cdots + \delta_q d_{t-q+1} + w_t, \tag{10.79}$$

where δ_i's are functions of β and ϕ_i's. Comparing (10.78) and (10.79) yields the following nonlinear restriction

$$\begin{aligned} \delta_1 &= \{1-\phi(\beta)\}^{-1} \\ \delta_j &= \delta\gamma(\beta)\{1-\delta\phi(\beta)\}^{-1}(\phi_{j+1} + \beta\phi_{j+2} + \cdots + \beta^p \phi_{j+p+1}) \text{ for } j = 2, \cdots, p. \end{aligned} \tag{10.80}$$

(?????$\gamma(\beta)$?) These are the nonlinear restrictions which (10.63) implies.

Masao needs to check this!

Example 10.1 Consider the case where d_t is an AR(1) process, so that $d_t = \phi_1 d_{t-1} + \nu_t$ where $|\phi_1| < 1$. Then $\hat{E}(d_{t+i}|\mathrm{H}_t) = \phi_1^i d_t$, and hence $\hat{E}(\sum_{i=1}^{\infty} \beta^i d_{t+i}|\mathrm{H}_t) = \sum_{i=1}^{\infty} \beta^i \phi_1^i d_t = \frac{\beta\phi_1}{1-\beta\phi_1} d_t$. Hence $p_t = \delta_1 d_t + w_t$ where $\delta_1 = \frac{\beta\phi_1}{1-\beta\phi_1}$. ∎

10.11.2 Econometric Methods

We focus on Hansen and Sargent's (1982) method which applies Hansen's (1982) Generalized Method of Moments (GMM) to linear rational expectations models.

Let $\mathbf{z}_{1t}$ be a vector of random variables in H_t. For example, $\mathbf{z}_{1t} = (d_t, \cdots, d_{t-q+1})'$. The unknown parameters β and ϕ_i's can be estimated by applying the GMM to orthogonality conditions $E(\mathbf{z}_{1t}\nu_{t+1}) = \mathbf{0}$ and $E(\mathbf{z}_{1t}w_t) = \mathbf{0}$ in the econometric system consisting of (10.75) and (10.79).

Let z_{2t} be a random variable in I_t, say d_t, and

$$p_t = \beta(p_{t+1} + d_{t+1}) + u_{t+1}. \tag{10.14}$$

[13] See West (1987), for the formula, which is based on Hansen and Sargent (1980), and on West (1988), for deterministic terms when d_t has a nonzero mean.

Then (10.62) implies another orthogonality condition $E(z_{2t}u_{t+1}) = 0$. This orthogonality condition can be used to estimate β. West (1987) forms a specification test *a la* Hausman (1978) by comparing the estimate of β from (10.14) with the estimate of β from (10.75) and (10.79). For this purpose, West forms a Wald test in the system consisting of (10.75), (10.79), and (10.14) without the restrictions (10.80) imposed. Another method to form West's specification test is to form a Lagrange Multiplier test or a likelihood ratio type test, which will require estimation constrained by the restrictions (10.80). This method may be preferable because of small sample problems with the Wald test for nonlinear restrictions (see Chapter 9 for discussions about these tests).

Some remarks are in order.

(A) Hansen and Sargent's method described above does not require an assumption that d_t is exogenous. Relation (10.75) or (**??**) is obtained from the assumption that d_t is covariance stationary and that its Wold representation is invertible.

(B) For the econometric system consisting of (10.75) and (10.79) (or (**??**) and (**??**)), random variables in H_t can be used as instruments, but the variables in I_t that are not in H_t are not valid instruments by construction.

(C) Since u_{t+1} in (10.14) is in I_{t+1} and ν_{t+1} in (10.75) is in H_{t+1}, u_{t+1} and ν_{t+1} are serially uncorrelated (see, e.g., Ogaki, 1993a, Section 6, for related discussions). However, w_t in (10.79) is not necessarily in H_{t+1}. Hence w_t has unknown order of serial correlation.

Masao needs to check this!

10.12 GMM for Consumption Euler Equations with Measurement Error

When data are contaminated by measurement error, the standard non-linear GMM yields inconsistent estimates (Garber and King, 1983; Amemiya, 1985). Such problem arises, for instance, in estimation of structural parameters in a non-linear consumption Euler equation when the consumption data contain measurement error.

To remedy this problem, Alan, Attanasio, and Browning (2005) propose two GMM estimators for consumption Euler equations in the presence of measurement error in data. Consider a simple life-cycle model with intertemporally additive and instantaneously iso-elastic utility. Under the assumption of rational expectations, a consumer's utility maximization yields the Euler equation,

$$E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\alpha} R_{t+1}\right] = 1, \tag{10.15}$$

where C^*_t is true consumption, R_{t+1} the gross real interest rate, α the coefficient of relative risk aversion, and $\beta < 1$ the discount factor. Call $\beta\left(C^*_{t+1}/C^*_t\right)^{-\alpha} R_{t+1}$ an expectational error uncorrelated with the time t information. We wish to estimate the preference parameters α and β. Suppose consumption data are observed with multiplicative error ϵ_t:

$$C_t = C^*_t \epsilon_t,$$

where C_t is the observed consumption. Assume that the measurement error is stationary, serially uncorrelated, and uncorrelated with C^*_t, R_t, and the expectational error for all t. Then, taking the expectations conditional on the time t information, we can write

(10.16)
$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{t+1}|\mathbf{I_t}\right]=E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\alpha}R_{t+1}|\mathbf{I_t}\right]E_t\left[\left(\frac{\epsilon_{t+1}}{\epsilon_t}\right)^{-\alpha}|\mathbf{I_t}\right]=\kappa,$$

where κ is a constant. The first equality follows from the assumption that the measurement error is independent of the expectational error, and the second equality follows from the Euler equation (16.11) and the stationarity assumption of the measurement error. For $\kappa \neq 1$, equation (16.57) implies that the standard GMM without consideration for the measurement error would result in inconsistent estimates of α and β. Similarly, consider the Euler equation representing the change in marginal utility between time t and $t+2$:

$$E_t\left[\beta^2\left(\frac{C_{t+2}}{C_t}\right)^{-\alpha}R_{t+1}R_{t+2}|\mathbf{I_t}\right]=\kappa.$$

Now define

$$\begin{aligned} u^1_{t+1} &\equiv \left[\beta\left(\tfrac{C_{t+1}}{C_t}\right)^{-\alpha}R_{t+1}-\kappa\right], \\ u^2_{t+2} &\equiv \left[\beta^2\left(\tfrac{C_{t+2}}{C_t}\right)^{-\alpha}R_{t+1}R_{t+2}-\kappa\right], \end{aligned} \tag{10.17}$$

where, by definition, u^1_{t+1} and u^2_{t+2} are uncorrelated with the time t information and $E_t(u^1_{t+1})=E_t(u^2_{t+2})=0$.

The first estimator, the *GMM-LN estimator*, additionally assumes that the measurement error is log-normally distributed with mean μ and variance σ^2. Let $u_{t+2}=[\ u^1_{t+1}\ \ u^2_{t+2}\ ]'$ and $z_t=[\ c\ \ z_{1t}\ ]'$ where c is a constant and z_{1t} is an instrument such as the lagged interest rate. Estimates of the parameters, α, β, and κ are obtained from four orthogonality conditions:

$$E\{u_{t+2}\otimes z_t\}=0. \tag{10.18}$$

Under the assumption of log-normality, κ can be written as

$$\kappa = \exp(\alpha^2\sigma^2). \tag{10.19}$$

Once α and κ are estimated using the orthogonality conditions (16.61), the estimate of the variance of measurement error σ^2 can be obtained from equation (16.58).

The second estimator, the *GMM-D estimator*, simply assumes stationarity and does not require any distributional assumption. Subtracting u^2_{t+2} from u^1_{t+1} in equations (16.60) yields

$$v_{t+2} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{t+1}\right] - \left[\beta^2\left(\frac{C_{t+2}}{C_t}\right)^{-\alpha} R_{t+1}R_{t+2}\right], \tag{10.20}$$

where v_{t+2} has zero mean and is independent of the time $t-1$ information. The orthogonality conditions for the GMM-D estimator are derived using equation (16.62) and a vector of instruments z_t. Note that because equation (16.62) takes the difference of the consumption growth (double-differencing), the GMM-D estimator is expected to be less precise than the GMM-LN estimator.[14]

Results from the Monte Carlo simulation in Alan, Attanasio, and Browning (2005) suggest that both proposed methods perform significantly better than conventional GMM estimators based on the log-linearized Euler equation or the exact Euler equation that ignores measurement error, especially when the panel length is short. In particular, both capture the true value of β remarkably well. They also report that when the measurement error is lognormally distributed, the distribution of α is more dispersed under the GMM-D estimator than under the GMM-LN estimator.

[14] In the presence of measurement error, the lagged consumption growth rate - a common choice for an instrument in estimation of consumption Euler equations - would be invalid since it is correlated with u_{t+2}. Instead, one should use the consumption growth rate with two-period lags. On the other hand, a one-period lag is sufficient for the interest rates since they are unlikely to be correlated with the measurement error (Alan, Attanasio, and Browning, 2005).

Exercises

10.1 (Computer Exercise) In the text we considered four alternative measures of the intertemporal marginal rate of substitution, m_t:

(i) $m_t = \beta(\frac{c_{t+1}}{c_t})^{-\gamma}$ (Hansen and Singleton)

(ii) $m_t = \beta^*(R_{t+1}^m)^{-\gamma}$ (Brown and Gibbons)

(iii) $m_t = \beta^+(R_{t+1}^m)^\eta(\frac{c_{t+1}}{c_t})^\theta$ (Epstein and Zin)

(iv) $m_t = \frac{\beta\{S_{t+1}^{-\gamma}+\beta a_1 s_{t+2}^{-\gamma}\}}{E(S_t^{-\gamma}+\beta a_1 s_{t+1}^{-\gamma}|I_t)}$ where $s_t = c_t + a_1 c_{t-1}$ (Ferson and Constantinides).

(**a**) For each of the four alternative measures, estimate the unknown parameters and test the overidentifying restrictions implied by the asset pricing relation $E(m_t R_{t+1}) = 1$. Use quarterly data on nondurables and services for consumption c_t, real value-weighted returns from the New York Stock exchange for R_t^m, and ex post real returns on Treasury Bill returns for R_t. Use a constant, one-period and two-period lagged values of $\frac{c_{t+1}}{c_t}$, and one-period and two-period lagged values of R_{t+1} for instrumental variables. You can modify the `GMM.EXP` file for models (i), (ii), and (iii), and `GMMHF.EXP` for model (iv). Note that `GMM.EXP` uses monthly data and `GMMHF.EXP` uses quarterly data. You will need to modify `GMM.EXP` to use the quarterly data used by `GMMHF.EXP`. For Ferson and Constantinides's, report results for both the truncated kernel and the non-prewhitened QS kernel.

For each measure, state what value "`mas`" should take in the GMM program and explain why. Comment on the relative strengths and weaknesses of the four measures of m_t from both a theoretical and an empirical perspective.

Print out the "`hu`" procedure part of your program and the final GMM iteration output for each model and submit them.

(**b**) Repeat the analysis in question (a) for the model (iii) using simultaneously the additional moment restrictions obtained by letting $R_t = R_t^m$. There is a difficulty in interpreting the empirical result for this case of multiple returns. What is the difficulty?

10.2 Let p_t be the real stock price, d_t be the real dividend, and β be the constant ex ante discount rate. Assume that p_t and d_t are stationary with zero mean and finite second moments. The stock price satisfies

$$p_t = \beta E(p_{t+1} + d_{t+1}|\mathrm{I}_t), \tag{10.E.1}$$

where I_t is the information set available at period t. We assume that I_t is generated from $\mathbf{x}_t, \mathbf{x}_{t-1}, \ldots$, where $\mathbf{x}_t$ is a random vector that includes p_t and d_t as its components. Solving (10.E.1) forward with the no bubble condition imposed, we obtain the present value formula:

$$p_t = \sum_{\tau=1}^{\infty} \beta^{\tau} E(d_{t+\tau}|\mathrm{I}_t) \tag{10.E.2}$$

Suppose that d_t is stationary with zero mean and finite second moments and let H_t be the information set generated by the linear functions of $\{d_t, d_{t-1}, d_{t-2}, \cdots\}$. Assume

$$\hat{E}(d_t|\mathrm{H}_{t-1}) = \phi d_{t-1}, \tag{10.E.3}$$

where $|\phi| < 1$, and $\hat{E}(\cdot|\mathrm{H}_{t-1})$ is the linear projection operator on H_t. Answer the following questions.

(a) Suppose that you run a regression

$$p_t = \delta d_t + w_t. \tag{10.E.4}$$

Your estimator for δ will converge to a number that can be expressed in terms of ϕ, and β. Derive this expression for δ. Show that $\hat{E}(w_t|\mathrm{H}_t) = 0$. Is it possible to prove that $E(w_t|\mathrm{I}_t) = 0$? Explain.

(b) Discuss whether or not w_t is serially correlated in general. If we make an additional assumption that p_t is in H_{t+1}, can you show that w_t is serially uncorrelated? Is this additional assumption realistic? Why?

(c) Explain how to use (10.E.4),

$$d_{t+1} = \phi d_t + v_{t+1}, \tag{10.E.5}$$

and

$$p_t = \beta(p_{t+1} + d_{t+1}) + u_t \tag{10.E.6}$$

to estimate β and ϕ in the framework of the Generalized Method of Moments, imposing the restriction on δ you derived. In particular, discuss the parameterized disturbances, valid instrumental variables, and appropriate methods to estimate the weighting matrix.

(d) List three tests that can be used to test the restriction on δ you derived. Discuss which tests may be better.

10.3 Let p_t be the log price level and m_t be the log money supply. A version of the Cagan's hyperinflation model assume that the demand for real money balance is

$$m_t - p_t = \alpha(E(p_{t+1}|\mathrm{I}_t) - p_t), \tag{10.E.7}$$

where I_t is the information set of the consumer, α is a negative number, and $-\alpha$ is the interest semi-elasticity of money demand when the real interest rate is assumed to be zero. Solving (10.E.7) as a difference equation for $E(p_{t+i})$ for a fixed t, and imposing the stability condition that the solution for p_t is bounded for all bounded sequences of m_t, we obtain

$$p_t = \frac{1}{1-\alpha} E\left(\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i m_{t+i} \middle| \mathrm{I}_t\right). \tag{10.E.8}$$

Suppose that m_t is stationary with zero mean and finite second moments (imagine that the data are already demeaned and detrended) and let H_t be the information set generated by the linear functions of $\{m_t, m_{t-1}, m_{t-2}, \cdots\}$. Assume

$$\hat{E}(m_{t+1}|\mathrm{H}_t) = \phi m_t, \tag{10.E.9}$$

where $|\phi| < 1$, and $\hat{E}(\cdot|\mathrm{H}_t)$ is the linear projection operator on H_t. Answer the following questions.

(a) Suppose that you run a regression

$$p_t = \delta m_t + w_t \tag{10.E.10}$$

Your estimator for δ will converge to a number that can be expressed in terms of ϕ, and α. Derive this expression for δ (note that the summation in (10.E.8) starts from $i = 0$ unlike West's present value model of the stock price in which the summation starts from $i = 1$).

(b) Discuss whether or not w_t is serially correlated in general. If we make an additional assumption that p_t is in H_{t+1}, can you show that w_t is serially uncorrelated? Is this additional assumption realistic? Why?

(**c**) Explain how to estimate α from the equation (10.E.7) with a time series data set on m_t and p_t.

(**d**) Explain how to use (10.E.9), (10.E.10), and

$$m_{t+1} = \phi m_t + v_{t+1} \tag{10.E.11}$$

to estimate α and ϕ in the framework of the Generalized Method of Moments, imposing the restriction on δ you derived in (i). In particular, discuss the parameterized disturbances, valid instrumental variables, and appropriate methods to estimate the weighting matrix.

(**e**) List three tests that can be used to test the restrictions on δ you derived in (i). Discuss which tests may be better.

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