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STRUCTURAL MACROECONOMETRICS

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PREFACE

This book presents various structural econometric tools used in macroeconomics. The word “structural” has been defined in many ways. In this book, “structural” means that explicit assumptions are made in econometric methods so that estimators or test statistics can be interpreted in terms of an economic model (or models) as explained in Chapter 1.

Many applied macroeconomists link macroeconomic models with econometric methods in this sense of structural econometrics. In principle, recent advances of theoretical time series econometrics make this task easier because they often relax the very restrictive assumptions made in conventional econometrics. There are many textbooks that explain these advanced econometric methods. It is often difficult, however, for applied researchers to exploit these advances because few textbooks in time series econometrics explain how macroeconomic models are mapped into advanced econometric models.¹ To fill this gap, this book presents methods to apply advanced econometric procedures to structural macroeconomic models. The econometric methods covered are mainly those of time series econometrics, and include the generalized method of moments, vector autoregressions, and estimation and testing in the presence of nonstationary variables.

Since this book focuses on applications, proofs are usually omitted with references given for interested readers. When proofs are helpful to understand issues that are important for applied research, they are given in mathematical appendices. Many examples are given to illustrate concepts and methods.

¹For example, Hamilton (1994) contains exceptional volume of explanations of applications for a time series econometrics textbook, but its main focus is on econometrics, and not on the mapping of economic models into econometric models.

This book is intended for an advanced graduate course in time series econometrics or macroeconomics. The prerequisites for this course would include an introduction to econometrics. This book is also useful to applied macroeconomic researchers interested in learning how recent advances in time-series econometrics can be used to estimate and test structural macroeconomic models.

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