Chapter 15

ECONOMIC MODELS AND COINTEGRATING REGRESSIONS

Economic models often imply cointegration. This chapter illustrates how to derive cointegration restrictions from economic models.

In order to derive cointegration restrictions, one must show that a linear combination of difference stationary random variables is stationary. This is often done by showing that a linear combination of difference stationary variables is a time independent function of a finite number of stationary random variables (see Proposition 2.2).

For many economic models, Proposition 2.2 can be directly used to show cointegration. In some other models, one more step is necessary. There are cases in which economic models imply that a linear combination of difference stationary variables is a conditional expectation of a variable. For example, suppose that an economic model implies

$$\mathbf{b}'\mathbf{y}_t = E(z_t|\mathrm{I}_t),$$

where z_t can be shown to be a stationary random variable because of Proposition

2.2. Here, I_t is typically the information set available to the economic agents, which includes both stationary and difference stationary random variables. Since z_t is stationary, $E(z_t|\mathrm{I}_t)$ is likely to be stationary. However, in order to formally show that $E(z_t|\mathrm{I}_t)$ is stationary, we need an additional assumption that $E(z_t|\mathrm{I}_t)$ is equal to $E(z_t|\mathrm{J}_t)$, where J_t is a subset of I_t and includes only a finite number of stationary variables. Then $E(z_t|\mathrm{J}_t)$ is a time independent function of a finite number of stationary variables, and we can use Proposition 2.2.

This additional assumption is not stringent as long as z_t is stationary. In order to see this property, suppose that I_t is generated by the current and past values of a difference stationary vector process $\mathbf{x}_t$:

$$\Delta\mathbf{x}_t = \mathbf{A}\Delta\mathbf{x}_{t-1} + \mathbf{u}_t.$$

where $\mathbf{u}_t$ is a vector i.i.d. white noise process and all roots of the characteristic equation lie outside the unit circle. Here $\mathbf{x}_t$ can include current and lagged values of many economic variables. If $z_t = \mathbf{c}'\Delta\mathbf{x}_{t+1}$, then

$$E(z_t|\mathrm{I}_t) = \mathbf{c}'\mathbf{A}\Delta\mathbf{x}_t = E(z_t|\Delta\mathbf{x}_t).$$

15.1 The Permanent Income Hypothesis of Consumption

The standard version of the permanent income hypothesis of consumption implies cointegration. The exact form of cointegration depends on the assumption on the form of the difference stationarity of labor income.

Consider a representative consumer who maximizes a quadratic utility function

$$E_t[\sum_{i=0}^{\infty}\beta^j(C_{t+i}-\gamma)^2], \tag{15.1}$$

subject to the budget constraint

$$A_{t+1} = (1+r)A_t + Y_t^l - C_t \tag{15.2}$$

and a no-Ponzi-game condition

$$\lim_{i \to \infty} (1+r)^i A_{t+i} = 0 \quad \text{almost surely.} \tag{15.3}$$

Here Y_t^l denotes labor income, and A_t is wealth at date t. Assuming that $\beta = (1+r)^{-1}$, the optimal consumption is

$$C_t = rA_t + (1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i Y_{t+i}^l]. \tag{15.4}$$

Substituting (15.4) back in the budget constraint, we obtain

$$A_{t+1} - A_t = Y_t^l - (1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i Y_{t+i}^l]. \tag{15.5}$$

Let $Y_t = rA_t + Y_t^l$ be total income, which includes both labor income and property income. Then (15.4) implies

$$C_t - Y_t = (1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i Y_{t+i}^l] - Y_t^l. \tag{15.6}$$

The cointegration implication of the permanent income hypothesis is different depending on whether we assume that the level of labor income is difference stationary or we assume that the log of labor income is difference stationary as pointed out by Cochrane and Sbordone (1988).

First, assume that the level of labor income is difference stationary so that $Y_t^l - Y_{t-1}^l$ is stationary. Then rewrite (15.6) as

$$C_t - Y_t = (1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i \{Y_{t+i}^l - Y_t^l\}]. \tag{15.6$'$}$$

Since the right hand side of (15.6′) is stationary, the permanent income hypothesis implies that $C_t - Y_t$ is stationary, which can be called the stationarity restriction. It remains to show that C_t and Y_t are difference stationary. From (15.5),

$$\begin{aligned} Y_{t+1} - Y_t &= -r(1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i \{Y^l_{t+i} - Y^l_t\}] \\ &+ Y^l_{t+1} - Y^l_t \end{aligned} \tag{15.7}$$

because the right hand side is stationary, the left hand side is stationary, and Y_t is difference stationary. The stationarity restriction implies that C_t is a sum of difference stationary Y_t and a stationary variable. Hence C_t is difference stationary. Therefore in this case, C_t and Y_t are cointegrated with a $(1, -1)$ cointegrating vector.

Second, assume that the log of labor income is difference stationary, so that $\ln(Y^l_t) - \ln(Y^l_{t-1})$ is stationary. Divide the both sides of (15.6) by Y_t to obtain

$$\frac{C_t}{Y_t} = 1 + (1-\beta)E_t[\sum_{i=0}^{\infty} \beta^i \frac{Y^l_{t+i}}{Y_t}] - \frac{Y^l_t}{Y_t}. \tag{15.8}$$

With an additional assumption that $\frac{Y^l_t}{A_t}$ is stationary, the right hand side of (15.8) is stationary, and $\ln(Y_t)$ and $\ln(C_t)$ are difference stationary. This additional assumption holds in standard general equilibrium models (see, e.g., King, Plosser, and Rebelo, 1988).

Thus, depending on whether the level of labor income or the log of labor income is assumed to be difference stationary, the permanent income hypothesis predicts different forms of cointegration. Which assumption is more appropriate? We observe from many economic data that the growth rate of an economic variable is stable over time. From this observation, it is more appropriate to assume that the log labor income is difference stationary rather than the level of labor income is difference stationary.

Another observation is that the assumption that the level of labor income is difference stationary implies that the level of saving, $Y_t - C_t$, is stationary. Since Y_t is nonstationary, the saving rate $\frac{Y_t - C_t}{Y_t}$ is nonstationary under this assumption. In contrast, the assumption that the log labor income is difference stationary implies that the saving rate, $\frac{Y_t - C_t}{Y_t} = 1 - \frac{C_t}{Y_t}$, is stationary. Hence this assumption is consistent with Kuznets' (1946) stylized fact that the saving rate is stable in the U.S. in the long-run.

15.2 Present Value Models of Asset Prices

The standard present value model implies that the stock price and the dividend are cointegrated. The exact form of cointegration implied by the model depends on whether the level or the log of the dividend is assumed to be difference stationary as pointed out by Cochrane and Sbordone (1988).

Let p_t be the real stock price (after the dividend is paid) in period t, and d_t be the real dividend paid to the owner of the stock in period t. Then the arbitrage condition is

$$p_t = E[b(p_{t+1} + d_{t+1})|\mathrm{I}_t], \tag{15.9}$$

where b is the constant real discount rate, and $E(\cdot|\mathrm{I}_t)$ is the mathematical expectation operator conditioned on the information set I_t in period t. Solving (15.9) forward and imposing the no bubble condition, we obtain the present value formula:

$$p_t = E(\sum_{i=1}^{\infty} b^i d_{t+i}|\mathrm{I}_t). \tag{15.10}$$

First, assume that d_t is difference stationary, following Campbell and Shiller

(1987). Then

$$p_t - \frac{b}{1-b}d_t = E[\sum_{i=1}^{\infty} b^i(d_{t+i} - d_t)|\mathrm{I}_t]. \tag{15.11}$$

Since $d_{t+i} - d_t$ is stationary for any i, the right hand side of (15.11) is stationary. Hence we obtain a stationarity restriction that $p_t - \frac{b}{1-b}d_t$ is stationary. This restriction implies that p_t is a sum of a difference stationary random variable and a stationary random variable. Hence p_t is difference stationary. This restriction also implies that p_t and d_t are cointegrated with a cointegrating vector $[1, -\frac{b}{1-b}]'$.

Second, assume that $\ln(d_t)$ is difference stationary. Then dividing both sides of (15.10) by d_t yields

$$\frac{p_t}{d_t} = E[\sum_{i=1}^{\infty} b^i \frac{d_{t+i}}{d_t}|\mathrm{I}_t]. \tag{15.12}$$

The right hand side of this equation is stationary. Hence, taking the log of both sides of (15.12), we obtain a stationarity restriction that $\ln(p_t) - \ln(d_t)$ is stationary. This restriction implies that $\ln(p_t)$ is a sum of a difference stationary random variable and a stationary random variable. Hence $\ln(p_t)$ is difference stationary. This restriction also implies that $\ln(p_t)$ and $\ln(d_t)$ are cointegrated with a cointegrating vector $(1, -1)'$.

When d_t is difference stationary, the cointegrating vector involves the discount factor, b. Hence a cointegrating regression can be used to estimate this structural parameter without making exogeneity assumptions. In addition to testing for cointegration, one can test the model by obtaining another estimate of b and compare it with a cointegrating regression estimate of b as explained in the next chapter. In contrast, when $\ln(d_t)$ is assumed to be difference stationary, the cointegrating vector is known, and no structural parameter can be estimated by a cointegrating regression. Even in this case, it is possible to test the model by testing for cointegration.

The near observational equivalence problem, however, tells us that cointegration test results are not reliable. Hence it is more interesting to assume that d_t is difference stationary than to assume $\ln(d_t)$ is difference stationary. Unfortunately, it is more reasonable to assume that $\ln(d_t)$ is difference stationary because the growth rates of the stock price and the dividends are relatively stable over time.

15.3 Applications to Money Demand Functions

Another application of cointegration is to assume directly that a demand or supply function is stable in the long run. The stable function can be estimated by a cointegrating regression, and the model can be tested by testing for cointegration. The most important application of this type is estimating a money demand function (see, e.g., Hoffman and Rasche, 1991; Stock and Watson, 1993).

Let M_t be the real money balance, Y_t be real income, and i_t be the nominal interest rate. Let the money demand function be

$$\ln(M_t) = a_0 + a_1 \ln(Y_t) + a_2 i_t + u_t. \tag{15.13}$$

If the money demand function is stable in the long run, u_t is stationary. If we assume that $\ln(Y_t)$ and i_t are difference stationary, and that they are not stochastically cointegrated, then the stable money demand function implies that $\ln(M_t)$ is difference stationary, and that $\ln(M_t), \ln(Y_t)$, and i_t are cointegrated with $(1, -a_1, -a_2)'$ as a cointegrating vector.

15.4 The Cointegration Approach to Estimating Preference Parameters

Ogaki and Park (1997) develop a cointegration approach to estimating preference parameters by utilizing the information in stochastic and deterministic time trends. The first order condition that equates the relative price and the contemporaneous marginal rate of substitution of two goods is used to derive the restriction that the relative price and consumption of the two goods are cointegrated.[1] The cointegrating vector involves preference parameters that are estimated with a cointegrating regression. In their application, they estimate the (long-run) intertemporal elasticity of substitution (IES) of nondurable consumption, which is a key parameter in a Consumption-Based Asset Pricing Model (C-CAPM). The parameter was also estimated by Hansen's (1982) GMM in a C-CAPM. The C-CAPM is rejected strongly by Hansen and Singleton (1982) when stock returns and Treasury Bill rates are used together. Possible reasons for the rejection of the C-CAPM have been pointed out. These include liquidity constraints (see, e.g., Hayashi, 1985; Zeldes, 1989), unknown preference shocks (e.g., Garber and King, 1983), time-nonseparable preferences (e.g., Eichenbaum, Hansen, and Singleton, 1988; Constantinides, 1990; Eichenbaum and Hansen, 1990; Ferson and Constantinides, 1991; Ferson and Harvey, 1992; Cooley and Ogaki, 1996; Heaton, 1995), and small information cost (Cochrane, 1998). GMM estimation of nonlinear Euler equations also assumes that there are no measurement errors.

Masao needs to check this!

[1] Ogaki and Park use Houthakker's (1960) addilog utility function. The cointegration approach can also be used to estimate the curvature parameters of the extended addilog utility function as in Atkeson and Ogaki (1996), and the CES utility function as in Ogaki and Reinhart (1998). Deaton and Wigley (1971), Deaton (1974), Miron (1986), and Ball (1990), among others, have estimated addilog utility functions. Ogaki (1988) introduces the cointegration approach to estimate preference parameters of the addilog utility function. Ogaki (1992) uses the cointegration approach to estimate income elasticities for food and other goods; Braun (1994), to estimate a utility function for cash and credit goods; Cooley and Ogaki (1996), to estimate a utility function for consumption and leisure; Amano and Wirjanto (1996) and Amano, Ho, and Wirjanto (1998) to estimate models of import demand; and Amano and Wirjanto (1997) to estimate a model of government spending. Working independently, Clarida (1994, 1996) estimates addilog utility functions to estimate price and income elasticities for imported goods with cointegrating regressions.

The cointegration approach provides an estimator that is consistent even in the presence of factors such as liquidity constraints, aggregation over heterogeneous consumers, unknown preference shocks, a general form of time-nonseparability, measurement errors, and the possibility that consumers do not know the true stochastic law of motion of the economy. The GMM estimator is not consistent in the presence of these factors, but the cointegrating regression estimator is consistent under certain assumptions. It is important to develop such an estimator because a great amount of recent research simulates economies with features that accounts for GMM's rejections of the C-CAPM such as liquidity constraints in recent works (see, e.g., Deaton, 1991; Marcet and Singleton, 1991; Heaton and Lucas, 1992). An estimator that is consistent in the presence of liquidity constraints can be used to guide the choice of parameters for these simulations.

In Section 15.5, we will discuss Cooley and Ogaki's (1996) test that compares the estimates obtained using cointegration techniques with those obtained using GMM in the spirit of Hausman's (1978) specification test. Since the GMM estimator is not consistent but the cointegrating regression estimator is consistent in the presence of factors such as liquidity constraints, this test can be interpreted as a test for the C-CAPM against an alternative hypothesis that such factors are present.

15.4.1 The Time Separable Addilog Utility Function

Suppose that a representative consumer maximizes the lifetime utility function[2]

$$U = E_0[\sum_{t=0}^{\infty} \beta^t u_t(C_t)] \tag{15.14}$$

[2]The existence of a representative consumer under complete markets was discussed by Ogaki (1997) for the general concave utility functions and by Atkeson and Ogaki (1996) for the extended addilog utility function. Ogaki and Park (1997) discussed a sufficient condition for aggregation under incomplete markets for the cointegration approach.

subject to a life time budget constraint in complete markets at period 0, where β is a discount factor and $C_t = (C_{1t}, C_{2t})$. Here C_{it} is real consumption of the i-th good, and $E_t(\cdot)$ denotes expectations conditional on the information available at period t. The intraperiod utility function is assumed to be of a monotone transformation of the addilog utility function:

$$u_t(C_t) = f_t(\sum_{i=1}^{2} \sigma_{it} \frac{C_{it}^{1-\alpha_i} - 1}{1 - \alpha_i}). \tag{15.15}$$

where $\alpha_i > 0$ for $i = 1, 2$. The stochastic process $\{\sigma_{1t}, \sigma_{2t}\}$, which is assumed to be (strictly) stationary, represents preference shocks. We refer to parameters α_1 and α_2 as curvature parameters. Nonseparability across goods is allowed by an arbitrary monotone transformation f_t with $f_t' > 0$.[3] This utility function includes Houthakker's (1960) addilog utility function and the Cobb-Douglas utility function ($\alpha_1 = \alpha_2 = 1$) as special cases. When $\alpha_1 \neq \alpha_2$, preferences are not homothetic.

Since time separability is assumed, a two-stage budgeting scheme can be applied to show that the consumer maximizes his/her intraperiod utility (15.15) subject to the intraperiod budget constraint

$$P_{1t}C_{1t} + P_{2t}C_{2t} = E_t, \tag{15.16}$$

where E_t is the total consumption expenditure at period t and P_{it} is the price of the i-th good. Let the first good be the numeraire for each period ($P_{1t} \equiv 1$).

The first order necessary conditions for the intraperiod optimization problem include

$$P_{2t} = \frac{\sigma_{2t} C_{2t}^{-\alpha_2}}{\sigma_{1t} C_{1t}^{-\alpha_1}}. \tag{15.17}$$

[3] Ogaki and Park (1992) showed that the cointegration approach allows for measurement errors, liquidity constraints, aggregation over heterogeneous consumers, and a general form of time-nonseparability in preferences. The present paper shows that the cointegration approach also allows for nonseparability across goods as long as time separability is assumed.

Since the first good is the numeraire, $P_{2t} = \frac{P_{2t}}{P_{1t}}$ is the relative price between the second good and the first good. Taking the natural logarithm of both side of (15.17) yields

$$p_{2t} - \alpha_1 c_{1t} + \alpha_2 c_{2t} = \ln(\frac{\sigma_{2t}}{\sigma_{1t}}) \quad (15.18)$$

where $p_{it} = \ln(P_{it}), c_{it} = \ln(C_{it})$. Thus the first order condition (15.17) implies a restriction that $p_2(t) - \alpha_1 c_{1t} + \alpha_2 c_{2t}$ be stationary. We call this restriction the stationarity restriction.

The stationarity restriction summarizes the long-run implication from the demand side. In order to model the supply side in the simplest way, let us consider an endowment economy without production. Let C^*_{it} be the endowment of the i-th good and $c^*_{it} = \ln(C^*_{it})$, so that $c_{it} = c^*_{it}$ in an equilibrium. In a production economy, we require that equilibrium consumption satisfies the trend properties of c^*_i that we assume. The trend properties of equilibrium consumption of the i-th good is likely to be closely related to those of the technology shock to the i-th good industry in a production economy. The stationarity restriction comes from an assumption of stable preference shocks in the long-run. Preference parameters can be identified from the stationarity restriction if the supply side exhibits much more volatility in the long-run than the demand side. This can be done by assuming that at least one of c^*_{1t} and c^*_{2t} has a stochastic trend. Stable preferences and technological shocks with stochastic trends seem to be plausible assumptions for identification.[4]

First, let us consider the case where both c^*_{1t} and c^*_{2t} are difference stationary:

Assumption 15.1a The process $\{c^*_{it} : t \geq 0\}$ is difference stationary for $i = 1, 2$.

[4]Ogaki (1988, 1989) showed that the Hansen and Singleton's (1982) GMM approach cannot be applied to the intraperiod first order condition of the addilog utility function when either c^*_{1t} or c^*_{2t} is difference stationary. For this reason, Ogaki (1988, 1989) assumed the trend stationarity for c^*_{1t} and c^*_{2t} to apply the GMM.

Assumption 15.1b The processes $\{c^*_{1t} : t \geq 0\}$ and $\{c^*_{2t} : t \geq 0\}$ are not stochastically cointegrated.

Assumption 15.1b will be satisfied for equilibrium consumption in a production economy if the technological shock in the second good industry has a different stochastic trend component from the technological shock in the food industry. Under assumption 15.1a and 15.1b, the stationarity restriction implies that the stochastic trends in $(p_{2t}, c_{1t}, c_{2t})'$ are eliminated by a cointegrating vector $(1, -\alpha_1, \alpha_2)$. The stationarity restriction also implies that the cointegrating vector eliminates the deterministic trends in $(p_{2t}, c_{1t}, c_{2t})'$. Thus the deterministic cointegration restriction will be satisfied under assumption 15.1a and 15.1b.

Second, consider the case where the log of the endowment of one good is difference stationary and that of the other good is trend stationary. There are two cases depending on which good is assumed to be trend stationary.

Assumption 15.2 The process $\{c^*_{1t} : t \geq 0\}$ is difference stationary, and the process $\{c^*_{2t} : t \geq 0\}$ is trend stationary with a nonzero trend.

Assumption 15.2′ The process $\{c^*_{1t} : t \geq 0\}$ is trend stationary with a nonzero trend, and the process $\{c^*_{2t} : t \geq 0\}$ is difference stationary.

Assumption 15.2 or Assumption 15.2′ will be satisfied for equilibrium consumption in a production economy if the technological shock in one good is trend stationary and the technological shock in the other good is difference stationary. For example, Costello (1990, chapter III) analyzed trend properties of Solow residuals of several industries and found some evidence that the Solow residual of the food industry is trend stationary and that of other industries is difference stationary.

Under assumption 15.2′, the stationarity restriction implies that p_{2t} and c_{2t} are stochastically cointegrated with a cointegrating vector $(1, \alpha_2)'$ and $(p_{2t}, c_{1t}, c_{2t})'$ is cotrended with a cotrending vector $(1, -\alpha_1, \alpha_2)'$. The curvature parameters can be identified from these conditions.

15.4.2 The Time Nonseparable Addilog Utility Function

The intra-period utility function is assumed to be of the addilog form

$$u_t = \sum_{i=1}^{n} \sigma_{it} \frac{S_{it}^{1-\alpha_i} - 1}{1 - \alpha_i}, \tag{15.19}$$

where $\alpha_i > 0$ for $i = 1, \cdots, n$ and σ_i's represent preference shocks. Here the stochastic process $\{(\sigma_{1t}, \cdots, \sigma_{nt})' : -\infty < t < \infty\}$ is assumed to be (strictly) stationary. This includes the case where some or all of σ_i's are constant. When α_i =1, we interpret $\frac{S_{it}^{1-\alpha_i}-1}{1-\alpha_i}$ to be $\ln(S_{it})$. Here S_{it} is the service flow from consumption purchases of good i. Purchases of consumption goods and service flows are related by

$$S_{it} = \{a_0^i C_{it} + a_1^i C_{i,t-1} + \cdots + a_k^i C_{i,t-k}\} \exp(\theta_i^s t) \tag{15.20}$$

for $i = 1, \cdots, n$, where C_{it} is real consumption expenditure for good i in period t. Following Eichenbaum and Hansen (1990), we allow for the possibility of technological progress in the transformation of purchases of good i into S_{it} in (15.20) via the exponential deterministic trend $\exp(\theta_i^s t)$. Below, we will consider the case in which the θ_i^s's are known to be zero as well as the case in which the θ_i^s's are unknown. Note that the purchase of one unit of good i at period t increases $S_{i,t+\tau}$ by $a_\tau^i \exp(\theta_{i,t+\tau}^s)$ units for nonnegative $\tau \leq k$. This type of method of specifying time-nonseparability is used by Hayashi (1982), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1990), and Heaton (1995), among others.

In our empirical work, we take a measure of nondurable consumption as one good (say good 1) and interpret the curvature parameter for nondurable consumption (α_1) as the long-run intertemporal elasticity of substitution (IES) for the consumption of nondurables.[5] As we will discuss in Section 2.4?????????????, this interpretation relies on the assumption of additive separability across the goods. It should be noted that this separability assumption is already made in Hansen and Singleton (1982) and Ferson and Constantinides (1991), both of which use the GMM approach and are closely related to this paper??????.

Masao needs to check this!

Masao needs to check this!

Let P_{it} be the purchase price of consumption good i. We take good 1 as a numeraire for each period: $P_{1t} \equiv 1$. The first order condition that equates the relative price between good i and good 1 ($P_{it} = \frac{P_{it}}{P_{1t}}$) with the marginal rate of substitution of these goods is

$$(15.21) \qquad P_{it} = \frac{\partial U/\partial C_{it}}{\partial U/\partial C_{1t}} = \frac{E_t[\sum_{\tau=0}^{k} \beta^{\tau} \frac{\partial u_{t+\tau}}{\partial C_{it}}]}{E_t[\sum_{\tau=0}^{k} \beta^{\tau} \frac{\partial u_{t+\tau}}{\partial C_{1t}}]} = \frac{E_t[\sum_{\tau=0}^{k} \beta^{\tau} \sigma_{i,t+\tau} a_{\tau}^{i} \exp(\theta_{i,t+\tau}^{s}) \{S_{i,t+\tau}\}^{-\alpha_i}]}{E_t[\sum_{\tau=0}^{k} \beta^{\tau} \sigma_{1,t+\tau} a_{\tau}^{1} \exp(\theta_{1,t+\tau}^{s}) \{S_{1,t+\tau}\}^{-\alpha_1}]}.$$

This first order condition forms the basis of the cointegration approach and summarizes the information needed from the demand side. In order to model the supply side in the simplest way, let us consider an endowment economy without production. Let C_{it}^{*} be the endowment of good i and $c_{it}^{*} = \ln(C_{it}^{*})$. In equilibrium, $c_{it} = \ln(C_{it}) = c_{it}^{*}$. In a production economy, we require that equilibrium consumption satisfies the trend properties we assume for c_{it}^{*}. The trend properties of equilibrium consumption are

[5]This parameter is the long-run IES for nondurable consumption when we allow current and past consumption to adjust. When preferences are time nonseparable, the short-run IES is different from the long-run IES because we take past consumption to be fixed in the short-run.

likely to be closely related to those of the technology shock to the good i industry in a production economy.

We consider three alternative assumptions about the trend properties of C^*_{it}. In each of the three assumptions, $\frac{C^*_{it}}{C^*_{i,t-1}}$ is stationary for all i. This assumption ensures that $\frac{S_{it}}{C_{it}\exp(\theta^s_i t)}$ is stationary in equilibrium. To see this property, let S^*_{it} be the S_{it} implied by C^*_{it} and note that $\frac{C^*_{i,t+\tau}}{C^*_{it}}$ is stationary for any fixed integer τ because $\frac{C^*_{i,t+\tau}}{C^*_{it}} = \frac{C^*_{i,t+\tau}}{C^*_{i,t+\tau-1}}\frac{C^*_{i,t+\tau-1}}{C^*_{i,t+\tau-2}}\cdots\frac{C^*_{i,t+1}}{C^*_{it}}$. It follows that the process $\{\frac{S^*_{i,t+\tau}}{C^*_{it}\exp(\theta^s_i t)} : -\infty < t < \infty\}$ is also stationary for any τ because the right hand side of

$$\begin{aligned}\text{(15.22)}\qquad \frac{S^*_{i,t+\tau}}{C^*_{it}\exp(\theta^s_i t)} &= \{a^i_0\frac{C^*_{i,t+\tau}}{C^*_{it}} + a^i_1\frac{C^*_{i,t+\tau-1}}{C^*_{it}} \\ &+ \cdots + a^i_k\frac{C^*_{i,t+\tau-k}}{C^*_{it}}\}\exp(\theta^s_i\tau)\end{aligned}$$

is stationary. We also make an extra assumption that the expectation of a stationary variable conditional on the consumer's information set is equal to the expectation conditional on the stationary variables included in his information set. Then

$$\frac{P_{it}\exp(\theta^s_1 t)[C^*_{1t}\exp(\theta^s_1 t)]^{-\alpha_1}}{\exp(\theta^s_i t)[C^*_{it}\exp(\theta^s_i t)]^{-\alpha_i}}$$

is stationary because the right hand side of

$$\begin{aligned}\text{(15.23)}\qquad &\frac{P_{it}\exp(\theta^s_1 t)[C^*_{1t}\exp(\theta^s_1 t)]^{-\alpha}}{\exp(\theta^s_i t)[C^*_{it}\exp(\theta^s_i t)]^{-\alpha_i}} \\ = &\frac{E_t[\sum_{\tau=0}^k \beta^\tau \sigma_{i,t+\tau} a^i_\tau \exp(\theta^s_i\tau)\{\frac{S^*_{i,t+\tau}}{C^*_{it}\exp(\theta^s_i t)}\}^{-\alpha_i}]}{E_t[\sum_{\tau=0}^k \beta^\tau \sigma_{1,t+\tau} a^1_\tau \exp(\theta^s_1\tau)\{\frac{S^*_{1,t+\tau}}{C^*_{1t}\exp(\theta^s_1 t)}\}^{-\alpha_1}]}\end{aligned}$$

is stationary. The right hand side of (15.23) is the ratio of conditional expectations of the functions of stationary variables.

Taking the natural log of the left hand side, we define z_t by

$$\text{(15.24)}\qquad z_t = p_{it} - \alpha_1 c^*_{1t} + \alpha_i c^*_{it} + (1-\alpha_1)\theta^s_1 t - (1-\alpha_i)\theta^s_i t$$

where $p_{it} = \ln(P_{it}), c^*_{it} = \ln(C^*_{it})$ for $i = 1, \cdots, n$ and conclude that z_t is stationary. We shall call this restriction as the stationary restriction. This restriction implies that $p_{it} - \alpha_1 c^*_{1t} + \alpha_i c^*_{it}$ $(= z_t - (1-\alpha_1)\theta^s_1 t + (1-\alpha_i)\theta^s_i t)$ is trend stationary in general, and is stationary if and only if $(1-\alpha_1)\theta^s_1 - (1-\alpha_i)\theta^s_i = 0$.

In this section, we study the implications of the stationarity restriction. We consider only the pair of good 1 and good 2 since our results generalize to any pair of goods. The stationarity restriction is a result of the assumption of the long-run stability of preferences. Preference parameters can be identified from the stationarity restriction if the supply side is substantially more volatile than the demand side in the long-run. This condition requires the assumption that at least one of c^*_{1t} and c^*_{2t} has a stochastic trend.[6] Stable preferences and technological shocks with stochastic trends seem to be plausible assumptions for identification.

First, consider the case in which both c^*_{1t} and c^*_{2t} are difference stationary:[7]

Assumption 15.3a The process $\{c^*_{it} : t \geq 0\}$ is difference stationary for $i = 1, 2$.

Assumption 15.3b The processes $\{c^*_{1t} : t \geq 0\}$ and $\{c^*_{2t} : t \geq 0\}$ are not stochastically cointegrated.

Assumption 15.3b will be satisfied for equilibrium consumption in a production economy if the technological shock in the good 1 industry has a different stochastic trend component from the technological shock in the good 2 industry. Under assumption 15.3a and 15.3b, the stationarity restriction implies that $p_{2t} - \alpha_1 c^*_{1t} + \alpha_2 c^*_{2t}$ is trend stationary. Thus $(p_{2t}, c^*_{1t}, c^*_{2t})'$ is stochastically cointegrated with a cointegrating vector

[6] Ogaki (1988) develops an econometric method based on GMM which uses the information in deterministic trends to estimate the preference parameters of the addilog utility function when both of c^*_{1t} and c^*_{2t} are trend stationary.

[7] A special case is that c^*_{1t} and c^*_{2t} are martingale when the real interest rate is constant and C^*_{it} is lognormally distributed.

$(1, -\alpha_1, \alpha_2)'$. However, the deterministic cointegration restriction is not necessarily satisfied under assumption 15.3a and 15.3b. The stationarity restriction implies that $p_{2t} - \alpha_1 c^*_{1t} + \alpha_2 c^*_{2t}$ is stationary under the condition that there is no technological progress in the transformation technology from consumption purchases to service flows (namely, $\theta^s_i = 0$ for $i = 1, 2$), Hence, consider the following assumption:

Assumption 15.4 Assumption 15.3a and 15.3b are satisfied and $\theta^s_i = 0$ for $i = 1, 2$.

Under assumption 15.4, $(p_{2t}, c^*_{1t}, c^*_{2t})'$ is stochastically cointegrated with a cointegrating vector $(1, -\alpha_1, \alpha_2)'$ and satisfies the deterministic cointegration restriction.

Second, consider the case where the log of the endowment of good 1 is difference stationary and that of good 2 is trend stationary:

Assumption 15.5a The process $\{c^*_{1t} : t \geq 0\}$ is difference stationary and the process $\{c^*_{2t} : t \geq 0\}$ is trend stationary with a nonzero linear trend.

Assumption 15.5b $\theta^s_i = 0$ for $i = 1, 2$.

Assumption 15.5a will be satisfied for equilibrium consumption in a production economy if the technological shock in the good 1 industry is difference stationary and the technological shock in the good 2 industry is trend stationary. Under assumption 15.5a, the stationarity restriction implies that p_{2t} and c^*_{1t} are stochastically cointegrated with a cointegrating vector $(1, -\alpha_1)'$. Assumption 15.5a is enough to identify α_1. In order to identify α_2 as well as α_1, we need assumption 15.5b. Under assumption 15.5a and 15.5b, the stationarity restriction implies that $(p_{2t}, c^*_{1t}, c^*_{2t})'$ is cotrended with a cotrending vector $(1, -\alpha_1, \alpha_2)'$.

15.4.3 Engel's Law and Cointegration

The key assumption for the cointegration approach to estimating preference parameters is that preferences are stable over time. Ogaki (1992) tests this assumption by comparing total expenditure elasticities (income elasticities in the context of the static models) estimated from time series data obtained by the cointegration approach with those estimated from household level cross-sectional data. The nonhomotheticity of preferences studied by Ogaki (1992) also important implications on intertemporal consumption decisions as in Atkeson and Ogaki (1996).

In cross sectional data, it is widely observed that a higher share of total expenditure goes to food for poorer households than is the case for richer households. A time series counterpart of this observation, Engel's law, is that the expenditure share on food declines as the economy grows. Ogaki (1992) tests if Houthakker's (1960) addilog utility function can explain both of these cross sectional and time series observations simultaneously. The cointegration approach is used to estimate parameters of the addilog utility function governing total expenditure elasticities of demand from time series data. Information in stochastic and deterministic trends is exploited in this approach.

Define $\mu = \frac{\partial \ln(C_{1t})}{\partial \ln(E(t))}$ as the total expenditure elasticity of demand for the first good, using the intraperiod optimization problem. It can be shown that the addilog utility function implies that the expenditure elasticity of demand for the first good is

$$\mu = \{\frac{\alpha_1}{\alpha_2} + \omega_{1t}(1 - \frac{\alpha_1}{\alpha_2})\}^{-1}, \tag{15.25}$$

where $\omega_{it} = \frac{P_{it}C_{it}}{E_t}$ is the budget share of the i-th good. Thus the expenditure elasticity for given levels of E_t, C_{1t}, and P_{2t} can be estimated once $\frac{\alpha_1}{\alpha_2}$ is estimated.

Comparing the expenditure elasticities implied by the addilog utility function estimated from the cointegration approach and the estimates of the elasticities estimated from cross-sectional household data provides a tests for the cointegration approach. The crucial assumption in the cointegration approach is that preferences are stable relative to the trends in equilibrium consumption expenditures. The most important factor that could cause problems with this assumption would probably be trending demographic changes. If this factor causes important problems, then the cointegration approach estimates from aggregate time series data will differ from the estimates from cross-sectional data.

Ogaki (1992) shows that the cointegration approach estimates of the expenditure elasticities from U.S. aggregate time series data are consistent with those from cross-sectional household data for food, clothing, household operation, and transportation. These empirical results support the assumption of stable preferences.

It should be noted that the expenditure elasticity is not constant. Suppose that $\alpha_1 > \alpha_2$, so that the first good is a necessary good. For very poor consumers, ω_{it} is close to one, and the elasticity is equal to one. For very rich consumers, ω_{it} is close to zero, and the elasticity is equal to $\frac{\alpha_2}{\alpha_1}$. When the relative price is constant, ω_{it} falls from one to zero as a consumer becomes richer, and the expenditure elasticity falls from one to $\frac{\alpha_2}{\alpha_1}$.

International comparison of elasticities is also of interest. Houthakker (1957) finds some tendency for the expenditure elasticity of the demand for food to be higher in low income countries than in high income countries. However, it seems important to allow for subsistence levels for low income countries. Atkeson and Ogaki (1996) estimate the extended addilog utility function, which generalizes the addilog utility function by allowing for subsistence levels:

$$u(C) = \sum_{i=1}^{n} \frac{\theta_i}{1-\alpha_i}[(C_i - \gamma_i)^{1-\alpha_i} - 1] \tag{15.26}$$

where $\alpha_i > 0$ and $\theta_i > 0$ for $i = 1, \cdots, n$. We refer to the parameters γ_i as subsistence parameters and the parameters α_i as curvature parameters. This utility function contains as special cases two utility functions commonly used in demand studies. If $\alpha_i = 1$ for $i = 1, .., n$, then this utility function yields the linear expenditure system in that the intratemporal demand functions for consumption of each good in excess of subsistence consumption are linear in expenditure in excess of subsistence expenditure. More generally, if $\alpha_1 = \alpha_2 = \cdots = \alpha_n$, then these preferences are quasi-homothetic. If $\gamma_i = 0$ for $i = 1, \cdots, n$, then this utility function is Houthakker's (1960) addilog utility function.

Atkeson and Ogaki (1996) discuss technical difficulties in estimating fixed subsistence levels from nonstationary time series data, and estimate them from Indian household panel data. The cointegration approach is applied to estimate the curvature parameters in Indian and U.S. aggregate time series data after factoring the estimated subsistence levels. They find little evidence against the hypothesis that preferences are identical for Indian and U.S. households when we maintain the hypothesis that the subsistence levels are the same for the two countries.

Houthakker (1957) finds that the expenditure elasticity of the demand for food is much lower for the typical Indian household than for the typical U.S. households in cross-sectional data. This finding can be consistent with identical preferences for Indian and U.S. households because the extended addilog utility function implies that the total expenditure elasticity of the demand for food will be different for rich and poorer households. Ogaki (1992) reports that the extended addilog utility function estimated by Atkeson and Ogaki (1996) explains the ratio of Houthakker's estimates of the elasticities for India and United States.

15.5 The Cointegration-Euler Equation Approach

This section explains Cooley and Ogaki's (1996) cointegration-Euler Equation approach, which combines the cointegration approach to estimating preference parameters with Hansen and Singleton's (1982) Euler equation approach based on GMM. In the first step of this approach, a cointegrating regression is applied to an intratemporal first order condition for the household's maximization problem to estimate some preference parameters. In the second step, GMM is applied to an Euler equation after plugging in point estimates from the cointegrating regression in the first step. Since the first step estimators are super consistent, asymptotic properties of the GMM estimators in the second step are not affected by the first step estimation.

This section explains Cooley and Ogaki's application of the approach on the consumption-leisure choice model for time nonseparable preferences that are additively separable for consumption and leisure. The next section explains Ogaki and Reinhart's (1998) application to estimate the intertemporal elasticity of substitution when preferences are nonseparable over nondurable and durable goods.

Cooley and Ogaki reexamine whether the time series properties of aggregate consumption, real wages, and asset returns are consistent with a simple neoclassical representative agent economy. Previous empirical explorations of this issue have rejected the neoclassical model in large part because the marginal rate of substitution between consumption and leisure does not equal the real wage as is implied by the first order conditions of the model. They argue that an optimal labor contract model is more appropriate for understanding the time series behavior of real wages and consumption. They show that a version of the optimal contract model restricts the long-run relation between real wages and consumption. They exploit this long-run restriction (cointegration restriction) to estimate preference parameters and test the model. First, they employ the cointegration approach to estimate the long-run intertemporal elasticity of substitution for nondurable consumption from a cointegrating regression. They test the model by testing for the cointegration restriction.

As further analysis, they use this estimated preference parameter in the asset pricing equation implied by this economy to estimate the discount factor and a coefficient of time-nonseparability using Hansen's (1982) Generalized Method of Moments (GMM). From this they are able to construct another specification test of the model.

Mankiw, Rotemberg, and Summers (1985, hereafter Mankiw *et al.*) subjected the Euler equations of an intertemporal labor supply model to a battery of tests and found no evidence to support it. Not only did their formal tests reject the model, but their point estimates of preference parameters implied a convex utility function. They concluded that the observed "$\cdots$ economic fluctuations do not easily admit of a neoclassical interpretation."

Eichenbaum, Hansen, and Singleton (1988, hereafter Eichenbaum *et al.*) also used the Euler equation approach, but their point estimates of preference parameters were more reasonable. They attributed their different finding to two factors. First, they removed trends by taking growth rates of variables and taking ratios of variables while Mankiw *et al.* did not. Second, Eichenbaum *et al.* allowed time-nonseparability of preferences. Though their point estimates were reasonable, their formal test statistics typically rejected the model at the one percent level when they tested both asset pricing equations and the first order condition that equates the real wage with the marginal rate of substitution between leisure and consumption. When they removed the first order condition and tested the asset pricing equations, their tests did not reject the model. However, the loss of precision of their estimates was substantial when the first order condition was removed. Eichenbaum *et al.* interpreted their results as suggesting that the optimal labor contract model might be appropriate for understanding real wages.[8]

A given Pareto optimal allocation can be consistent with a wide variety of institutional arrangements. In optimal labor contract models (see, e.g, Azariadis, 1975; Rosen, 1985; Wright, 1988), labor income contains a component that provides workers with some degree of protection against business cycle fluctuations (also see Hall, 1980). This insurance component of labor income inserts a wedge between the marginal rate of substitution between leisure and consumption and wages. In their empirical work, Gomme and Greenwood (1995) showed that accounting for this component could help explain the observed pattern of fluctuations in income. These arguments combined with the findings of Eichenbaum *et al.* suggest that the imposition of the requirement that wages equal the marginal rate of substitution between consumption and leisure is too confining.

Cooley and Ogaki use a restriction on the time series properties of real wages and consumption that is implied by optimal labor contract to estimate preference parameters and test the model. In the optimal contract model, the first order condition for real wages and consumption does not hold on a period-by-period basis.

[8]Osano and Inoue (1991) used an approach similar to Eichenbaum *et al.* to test the overidentifying restrictions of Euler equations, using aggregate Japanese data. They also noted that there was much less evidence against the model when they removed the Euler equation associated with the equation of real wages and the marginal product of labor.

They show, however, that a version of the optimal contract model implies that the real wage rate is equated with the marginal rate of substitution between consumption and labor in the long- run. They exploit this long-run restriction for estimating and testing the model.

In contrast to the research cited above, the cointegration approach yields results that are supportive of the representative agent model. In the first step of our econometric procedure, we test the null hypothesis of cointegration and estimate the long-run IES for three measures of nondurable consumption. Cooley and Ogaki do not reject the null of cointegration and obtain reasonable estimates. The long-run IES appears in the asset pricing equation derived from the representative consumer model. Cooley and Ogaki use the estimated IES parameter from the cointegrating regression in the first step in the asset pricing equation and apply GMM to estimate the discount parameter and a coefficient of time-nonseparability. They use both stock and nominal risk free returns. They form a specification test *a la* Hausman (1978) through these steps. This specification test does not reject the model.

15.5.1 The Economy

We consider an economy populated by N households who have preferences defined over consumption and the flow of services from their leisure time. Each household maximizes

$$U = E_0[\sum_{t=0}^{\infty} \beta^t u_t] \tag{15.27}$$

where E_t denotes the expectation conditioned on the information available at t. In order to develop intuition, let us first consider a simple intraperiod utility function that is assumed to be time- and state-separable and separable in nondurable consumption, durable consumption, and leisure

$$u_t = \frac{C_t^{1-\alpha} - 1}{1-\alpha} + v(l_t) \tag{15.28}$$

where $v(\cdot)$ is a continuously differentiable concave function, C_t is nondurable consumption, and l_t is leisure.

For now, assume that real wages do not contain any insurance component. Then the usual first order condition for a household that equates the real wage rate with the marginal rate of substitution between leisure and consumption is:

$$W_t = \frac{v'(l_t)}{C_t^{-\alpha}} \tag{15.29}$$

where W_t is the real wage rate. We assume that the stochastic process of leisure is (strictly) stationary in the equilibrium as in Eichenbaum, Hansen, and Singleton (1988) and that the random variables used to form the conditional expectations for stationary variables are stationary. Then an implication of the first order condition is that $\ln(W_t) - \alpha \ln(C_t) = \ln(v'(l_t))$ is stationary. When we assume that the log of consumption is difference stationary, this assumption implies that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\alpha)'$. We exploit this cointegration restriction to identify the curvature parameter α from cointegrating regressions.

Given that the saving rate is stable in the long-run in the U.S. (as Kuznets, 1946, found), it is natural to impose a restriction that the ratio of total consumption expenditure and labor income is stable at least when a consumer is rich enough. Since we assume that consumption and leisure are additively separable in intertemporal preferences, this restriction implies that α is equal to one when total consumption expenditure is used as C_t in our model. In our empirical work, we use a measure of nondurable consumption as C_t, assuming that the other consumption goods (say, durable consumption goods) are additively separable from the measure of nondurable consumption good used in our analysis. For this reason, α can be different from one even when the saving rate is stationary for rich enough consumers.[9]

Masao needs to check this!

We now introduce time-nonseparability of preferences. The intraperiod utility function is assumed to be

$$u_t = \frac{S_t^{1-\alpha} - 1}{1-\alpha} + v(l_t, l_{t-1}, ..., l_{t-k}), \tag{15.30}$$

where S_t is the service flow from nondurable consumption:

$$S_t = C_t + \lambda C_{t-1}. \tag{15.31}$$

This type of time nonseparable specification of leisure has been used by many authors and is useful because it can capture the fact that households may use leisure time in a household production technology to augment a stock of household capital (Kydland, 1984; Greenwood and Hercowitz, 1991; Benhabib, Rogerson, and Wright, 1991).

The time-nonseparable specification for nondurable consumption is similar to that considered by Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1990), Constantinides (1990), Heaton (1993, 1995), Allen (1992), and Braun, Constantinedes, and Ferson (1993) among others, except that some of these authors considered a more general form of time-nonseparability for nondurable consumption than (15.31). We have habit formation for nondurable consumption when λ is neg-

[9] Since many economic models imply known cointegrating vectors when the log of the variables are taken and because an attractive feature of cointegration is that unknown parameters can be estimated without exogeneity assumptions, the fact that α is unknown in the model is important. For this reason, this point that α can be different from one is explained in details in the Appendix.??????????????

ative and local substitutability or durability when λ is positive.[10] Note that the time-nonseparability does not affect the IES in the long-run when C_t and C_{t-1} are equal.[11] We will refer to $\frac{1}{\alpha}$ as the long-run IES for nondurable consumption.

The usual first order condition for a household that equates real wage rate with the marginal rate of substitution between leisure and consumption is now:

$$W_t = \frac{\partial U/\partial l_t}{\partial U/\partial C_t} = \frac{E_t[\sum_{\tau=0}^{K}\beta^\tau \frac{\partial u_{t+\tau}}{\partial l_t}]}{E_t[\frac{\partial u_t}{\partial C_t} + \frac{\partial u_{t+1}}{\partial C_t}]} = \frac{E_t[\sum_{\tau=0}^{K}\beta^\tau \frac{\partial v_{t+\tau}}{\partial l_t}]}{E_t[S_t^{-\alpha} + \beta\lambda S_{t+1}^{-\alpha}]}. \tag{15.32}$$

We assume that $\ln(C_t)$ is difference stationary in the equilibrium. Then

$$\frac{S_{t+\tau}}{C_t} = \frac{C_{t+\tau}}{C_t} + \lambda\frac{C_{t+\tau-1}}{C_t} \tag{15.33}$$

is stationary for any τ. Combined with the first order condition (15.32), it follows that

$$W_t C_t^{-\alpha} = \frac{E_t[\sum_{\tau=0}^{K}\beta^\tau \frac{\partial v_{t+\tau}}{\partial l_t}]}{E_t[\{\frac{S_t}{C_t}\}^{-\alpha} + \beta\lambda\{\frac{S_{t+1}}{C_t}\}^{-\alpha}]} \tag{15.34}$$

is stationary. Taking logs, $\ln(W_t) - \alpha\ln(C_t)$ is stationary as in the time-separable case we discussed.

In Cooley and Ogaki's empirical work, they estimate and test the first order condition (15.32) through the cointegration restriction for aggregated real wages and consumption. They also estimate and test the standard asset pricing equation for the

[10] The time-nonseparability for nondurable consumption allows us to separate the IES in the short-run and the reciprocal of the RRA coefficient as Constantinides (1990) described, which could help explain the equity premium puzzle of Mehra and Prescott (1985). Ferson and Constantinides (1991) found evidence in favor of the asset pricing model with habit formation, using GMM.

[11] Alternatively, C_t grows at a constant rate in the long-run.

time-nonseparable utility function

$$\frac{E_t[\beta\{S_{t+1}^{-\alpha} + \beta\lambda S_{t+2}^{-\alpha}\}R_{t+1}]}{E_t[S_t^{-\alpha} + \beta\lambda S_{t+1}^{-\alpha}]} = 1 \tag{15.35}$$

for any gross asset return R_t.

In optimal labor contract models, labor income contains a component that provides workers with some degree of protection against business cycle fluctuations. This insurance component of labor income inserts a wedge between the marginal rate of substitution between leisure and consumption and wages. To utilize information in the first order condition (15.32) for estimation and testing, we start from the observation that the cointegration restriction is robust as long as the measured wage rate has the same trend as the marginal rate of substitution. Even when there is a wedge between the real wage rate and the marginal rate of substitution, the stationary restriction holds as long as the insurance component does not have (stochastic or deterministic) trends. Intuition suggests that the fraction of the insurance component in the wage rate is likely to be stationary rather than trending. Cooley and Ogaki formalize this intuition by considering a version of an optimal contract model.

15.5.2 The 2-Step Estimation Method

In the first step, a cointegration regression is used to estimate α from the stationarity restriction. Since the log real wage rate and log consumption are cointegrated, either variable can be used as a regressand. In finite samples, the empirical results will be different depending on the choice of the regressand. However, the results should be approximately the same as long as cointegration holds and the sample size is large enough.

The econometric model for our GMM procedure is based on the asset pricing equation (15.35), which implies $E_t(\epsilon^0_{gt}) = 0$, where

$$\epsilon^0_{gt} = \beta[(C_{a,t+1} + \lambda C_{a,t})^{-\alpha} + \lambda\beta(C_{a,t+2} + \lambda C_{a,t+1})^{-\alpha}]R_{t+1} - [(C_{a,t} + \lambda C_{a,t-1})^{-\alpha} + \lambda\beta(C_{a,t+1} + \lambda C_{a,t})^{-\alpha}] \quad (15.36)$$

where C_a indicates aggregate nondurable consumption. We define

$$\epsilon_{gt} = \frac{\epsilon^0_{gt} g(\lambda)}{(1+\beta\lambda)\{C_{a,t} + \lambda C_{a,t-1}\}^{-\alpha}}$$

where $g(\lambda) = 1$ if $\lambda \leq 1$ and $g(\lambda) = 1 + (\lambda - 1)^2$ if $\lambda > 1$. We use ϵ_{gt} as the disturbance for the GMM estimation. Since the scale factor $\frac{g(\lambda)}{(1+\beta\lambda)\{C_{a,t}+\lambda C_{a,t-1}\}^{-\alpha}}$ is in the information available at t, $E_t(\epsilon_{gt}) = 0$. We scale the disturbance to achieve stationarity required for the GMM,[12] to avoid the trivial solutions that cause an identification problem, and to incorporate the prior information that λ is likely to be smaller than one in absolute value.[13] Even though the asymptotic theory justifies this type of scaling, small sample properties of the GMM estimator are affected by the choice of the scaling factor. For this reason, the $g(\lambda)$ function is designed not to affect the disturbance when $\lambda \leq 1$: we have little prior information about which admissible value of λ is more plausible when the absolute value of λ is less than one. The disturbance term is MA of order one because of the time-nonseparable specification. The weighting matrix for the GMM estimation must take account of the serial correlation.

A formal test statistic can be formed by using the estimate of α from the cointegrating regression in the GMM procedure to obtain restricted estimates. In this

[12]The stationarity assumption of the GMM can be relaxed to some extent, but unit-root nonstationarity is not allowed. Hence the stationary inducing transformation is necessary for our model.

[13]Certain values of λ are not admissible because $C_{a,t} + \lambda C_{a,t-1}$ cannot be negative. In order to exclude these values in the GMM nonlinear search, a very large positive number was returned as ϵ_{gt} when they are tried. The numerical derivative program was modified accordingly. See Ogaki (1993b) for details.

restricted GMM estimation, we estimate only β and λ. We use the same weighting matrix to form unrestricted estimates. We then take the difference of Hansen's (1982) chi-square test (Hansen's J_T test) statistic for the overidentifying restrictions from the restricted estimation and that from the unrestricted estimation, in which β, λ, and α are estimated. The difference is the likelihood ratio type test (denoted by C_T), which has an asymptotic chi-square distribution with one degree of freedom.[14] This two step procedure does not alter the asymptotic distribution of GMM estimators and test statistics because our cointegrating regression estimator is super consistent and converges at a rate faster than $\sqrt{T}$.

15.5.3 Measuring Intertemporal Substitution: The Role of Durable Goods

Masao needs to check this!

[To Be added??????]

15.6 Purchasing Power Parity

Masao needs to check this!

[?????? This section is incomplete]

Assume that there is only one good in the world economy, and that the law of one price for the word economy (called Purchasing Power Parity (PPP)) holds at each point in time. Let P_t be the domestic price of the good, and P_t^F be the foreign price of the good at t. Define the *real exchange rate* as

$$S_t^r = \frac{S_t P_t^F}{P_t} \qquad (15.37)$$

When the good is measured with the same unit in the two countries, PPP implies that the real exchange rate is equal to one. This version of PPP is called *absolute* PPP.

[14] See, e.g., Ogaki (1993a) for an explanation of the likelihood ratio type test for GMM.

When the good is measured with different units in the two countries, PPP implies that the real exchange rate is constant. This version of PPP is called *relative* PPP.

Two cases are worth noting. First, if infinitely many stationary random variables are involved in an economic model, it is often possible to show that an infinite sum of a series of random variables (or vectors) converges to a stationary random variable (or vector). Then it is possible to use Proposition ??.??.

Masao needs to check this!

Exercises

15.1 Suppose that a representative consumer maximizes the life time utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u_t \tag{15.E.1}$$

at time 0, where $E_t(\cdot)$ denotes expectations conditional on the information available at time t, I_t, subject to a life time budget constraint in an Arrow-Debreu economy with two goods. The intra-period utility function is assumed to be

$$u_t = \frac{C_{1t}^{1-\alpha_1} - 1}{1-\alpha_1} + \sigma_2 \frac{S_{2t}^{1-\alpha_2} - 1}{1-\alpha_2} \tag{15.E.2}$$

where $\alpha_i > 0$ for $i = 1, 2$ and

$$S_{2t} = e^{\theta t}(C_{2t} + \delta C_{2,t-1}) \tag{15.E.3}$$

is service flow from purchases of the second consumption good. Let P_{2t} be the purchasing price of the second good in terms of the first good and R_t be the ex post gross rate of return of an asset in terms of the second good. Assume that $\{\frac{C_{i,t+1}}{C_{it}}\}$ is stationary for $i = 1, 2$.

(**a**) Write down the parametric form of the first order condition that $p_{2t}, C_{1t}, C_{2,t-1}, C_{2t}$ and $C_{2,t+1}$ should satisfy in an equilibrium. Explain your answer.

(**b**) Show that $ln\frac{S_{2t}}{C_{2t}}$ is trend stationary.

(**c**) Give the definitions of stochastic cointegration and the deterministic cointegration restriction. In each of the following cases, which variables are stochastically cointegrated? Give a cointegrating vector and explain whether or not the deterministic cointegration restriction is satisfied for these variables that are stochastically cointegrated. Explain your answers.

Case 1: $\theta = 0$ and lnC_{it} is difference stationary for $i = 1, 2$.

Case 2: $\theta \neq 0$ and lnC_{it} is difference stationary for $i = 1, 2$.

Case 3: $\theta = 0$ and lnC_{1t} is difference stationary, and lnC_{2t} is stationary.

Case 4: $\theta \neq 0$ and lnC_{1t} is difference stationary, and lnC_{2t} is stationary.

Case 5: $\theta = 0$ and lnC_{1t} is difference stationary, and lnC_{2t} is trend stationary with a nonzero linear trend.

Case 6: $\theta \neq 0$ and lnC_{1t} is difference stationary, and lnC_{2t} is trend stationary with a nonzero linear trend.

15.2 Take nondurables as the first good and durables in the national account as the second good in Ogaki's (1992) model. To obtain per equivalent adult consumption, place an weight of 1 for the civilian noninstitutional population with ages 16 and over, and 0.55 on the rest of the total population. The consumption data are in `QNRND91.DAT`(nondurables) and `QNRD91.DAT`(durables). These files also include data descriptions in detail. These quarterly data files contain the current dollar consumption in the first column and the 1987 dollar consumption in the second column over the period 1947:1-1993:4. The population data are in `MPOP92.DAT` which contains the total population in the first column and the total civilian noninstitutional population with ages 16 and over in the second column. This monthly data covers 1947:1-1992:1. Take the quarterly average of the equivalent adult population. Use the sample period 1947:2-1989:4. All the files necessary for this exercise are in *http://economics.sbs.ohio-state.edu/ogaki*. You can modify and rename `*.EXP` file for each of the `*.SET` file mentioned in the problems. Imagine that you were reporting your empirical results in a section of a paper to be published in a professional journal. Report results in tables and explain purposes of tests and your results.

(**a**) Report $G(1,q)$ test statistics with $q = 2,3$ for lnC_{1t} and lnC_{2t} with non-prewhitened QS kernel. Use `GPQ.SET`.

(**b**) Report augmented Dickey-Fuller (Said-Dickey) test statistics for lnC_{1t} and lnC_{2t}.

(**c**) Report the third stage CCR estimators for preference parameters with lnC_{1t} as the regressand. Also report $H(0,1)$, $H(1,q)$ test statistics with $q = 2,3$, and Wald test statistics for the null hypothesis $\alpha_1 = \alpha_2 = 1$ from the fourth stage CCR with the singular values for the prewhitening VAR matrix bounded by 0.99 and the automatic bandwidth parameter bounded by $\sqrt{T}$ where T is the sample size. Use `CCR.SET`.

References

Allen, E. R. (1992): "Cross Sectional and Time Series Measures of Asset Pricing Model Fit," Manuscript, University of Huston.

Amano, R. A., W.-M. Ho, and T. S. Wirjanto (1998): "Intraperiod and Intertemporal Substitution in Import Demand," Bank of Canada, CREFE Working paper No. 84.

Amano, R. A., and T. S. Wirjanto (1996): "Intertemporal Substitution, Imports, and the Permanent-Income Model," *Journal of International Economics*, 40, 439–457.

——— (1997): "Intratemporal Substitution and Government Spending," *Review of Economics and Statistics*, 40, 605–609.

Atkeson, A., and M. Ogaki (1996): "Wealth-Varying Intertemporal Elasticities of Substitution: Evidence from Panel and Aggregate Data," *Journal of Monetary Economics*, 38, 507–534.

Azariadis, C. (1975): "Implicit Contracts and Unemployment Equilibria," *Journal of Political Economy*, 83, 1183–1202.

Ball, L. (1990): "Intertemporal Substitution and Constraints on Labor Supply - Evidence from Panel Data," *Economic Inquiry*, 28(4), 706–724.

Benhabib, J., R. Rogerson, and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99, 1166–1187.

Braun, P. A., G. M. Constantinedes, and W. E. Ferson (1993): "Time Nonseparability in Aggregate Consumption: International Evidence," *European Economic Review*, 37, 897–920.

Braun, R. A. (1994): "How Large is the Optimal Inflation Tax?," *Journal of Monetary Economics*, 34, 201–214.

Campbell, J. Y., and R. J. Shiller (1987): "Cointegration and Tests of Present Value Models," *Journal of Political Economy*, 95(5), 1062–1088.

Clarida, R. H. (1994): "Cointegration, Aggregate Consumption, and the Demand for Imports: A Structural Econometric Investigation," *American Economic Review*, 84(1), 298–308.

——— (1996): "Consumption, Import Prices, and the Demand for Imported Consumer Durables: A Structural Econometric Investigation," *Review of Economics and Statistics*, 78, 369–374.

Cochrane, J. H. (1998): "What Do the VARs Mean? - Measuring the Output Effects of Monetary Policy," *Journal of Monetary Economics*, 41, 277–300.

Cochrane, J. H., and A. M. Sbordone (1988): "Multivariate Estimates of the Permanent Components of GNP and Stock Prices," *Journal of Economic Dynamics and Control*, 12, 255–296.

Constantinides, G. M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98, 519–543.

Cooley, T. F., and M. Ogaki (1996): "A Time Series Analysis of Real Wages, Consumption, and Asset Returns," *Journal of Applied Econometrics*, 11(2), 119–134.

Costello, D. M. (1990): "Productivity Growth, the Transfer of Technology and International Business Cycles," Ph.D dissertation, University of Rochester.

Deaton, A. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59(5), 1221–1248.

Deaton, A. S. (1974): "The Analysis of Consumer Demand in the United Kingdom, 1900–1970," *Econometrica*, 42, 341–367.

Deaton, A. S., and K. K. Wigley (1971): "Econometric Models for the Personal Sector," *Bulletin of the Oxford Institute of Statistics*, 33.

Eichenbaum, M., and L. P. Hansen (1990): "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," *Journal of Business and Economic Statistics*, 8, 53–69.

Eichenbaum, M., L. P. Hansen, and K. J. Singleton (1988): "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty," *Quarterly Journal of Economics*, 103, 51–78.

Ferson, W. E., and G. M. Constantinides (1991): "Habit Persistence and Durability in Aggregate Consumption: Empirical Tests," *Journal of Financial Economics*, 29(2), 199–240.

Ferson, W. E., and C. R. Harvey (1992): "Seasonality and Consumption-Based Asset Pricing," *Journal of Finance*, 47, 511–552.

Garber, P. M., and R. G. King (1983): "Deep Structural Excavation? A Critique of Euler Equation Methods," NBER Technical Working Paper No. 31.

Gomme, P., and J. Greenwood (1995): "On the Cyclical Allocation of Risk," *Journal of Economic Dynamics and Control*, 19(1–2), 91–124.

Greenwood, J., and Z. Hercowitz (1991): "The Allocation of Goods and Time over the Business Cycle," *Journal of Political Economy*, 99, 1188–1214.

Hall, R. E. (1980): "Labor Supply and Aggregate Fluctuations," *Carnegie-Rochester Conference Series on Public Policy*, 12, 7–33.

Hansen, L. P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50(4), 1029–1054.

Hansen, L. P., and K. J. Singleton (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50(5), 1269–1286.

Hausman, J. A. (1978): "Specification Tests in Econometrics," *Econometrica*, 46(6), 1251–1271.

Hayashi, F. (1982): "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables," *Journal of Political Economy*, 90, 895–916.

——— (1985): "The Effect of Liquidity Constraints on Consumption: A Cross-Sectional Analysis," *Quarterly Journal of Economics*, 100, 183–206.

Heaton, J. C. (1993): "The Interaction Between Time-Nonseparable Preferences and Time Aggregation," *Econometrica*, 61, 353–385.

——— (1995): "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications," *Econometrica*, 63, 681–717.

Heaton, J. C., and D. Lucas (1992): "The Effects of Incomplete Insurance Markets and Trading Costs in a Consumption-Based Asset Pricing Model," *Journal of Economic Dynamics and Control*, 16(3–4), 601–620.

Hoffman, D. L., and R. H. Rasche (1991): "Long-Run Income and Interest Elasticities of Money Demand in the United States," *Review of Economics and Statistics*, 73(4), 665–674.

Houthakker, H. S. (1957): "An International Comparison of Household Expenditure Patterns, Commemorating the Centenary of Engel's Law," *Econometrica*, 25(4), 532–551.

——— (1960): "Additive Preferences," *Econometrica*, 28(2), 244–257.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988): "Production, Growth and Business Cycles: II. New Directions," *Journal of Monetary Economics*, 21, 309–341.

Kuznets, S. (1946): *National Income: A Summary of Findings*. National Bureau of Economic Research, New York.

Kydland, F. E. (1984): "Labor-Force Heterogeneity and the Business Cycle," *Carnegie-Rochester Conference Series on Public Policy*, 21, 173–208.

Mankiw, N. G., J. J. Rotemberg, and L. H. Summers (1985): "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, 100(1), 225–251.

Marcet, A., and K. J. Singleton (1991): "Optimal Consumption-Savings Decisions and Equilibrium Asset Prices in A Model with Heterogeneous Agents Subject to Portfolio Constraints," Manuscript, Stanford University.

Mehra, R., and E. C. Prescott (1985): "The Equity Premium: a Puzzle," *Journal of Monetary Economics*, 15, 145–161.

Miron, J. A. (1986): "Seasonal Fluctuations and the Life-Cycle Permanent Income Model of Consumption," *Journal of Political Economy*, 94(6), 1258–1279.

Ogaki, M. (1988): "Learning about Preferences from Time Trends," Ph.D. thesis, University of Chicago.

——— (1989): "Information in Deterministic Trends about Preferences," Manuscript.

——— (1992): "Engel's Law and Cointegration," *Journal of Political Economy*, 100(5), 1027–1046.

——— (1993a): "Generalized Method of Moments: Econometric Applications," in *Handbook of Statistics: Econometrics*, ed. by G. S. Maddala, C. R. Rao, and H. D. Vinod, vol. 11, chap. 17, pp. 455–488. North-Holland, Amsterdam.

——— (1993b): "Unit Roots in Macroeconometrics: A Survey," *Monetary and Economic Studies*, 11(2), 131–154.

——— (1997): "Aggregation under Complete Markets," Working Paper No. 97-05, Department of Economics, Ohio State University.

Ogaki, M., and J. Y. Park (1992): "A Cointegration Approach to Estimating Preference Parameters," Manuscript.

——— (1997): "A Cointegration Approach to Estimating Preference Parameters," *Journal of Econometrics*, 82(1), 107–134.

Ogaki, M., and C. M. Reinhart (1998): "Measuring Intertemporal Substitution: The Role of Durable Goods," *Journal of Political Economy*, 106, 1078–1098.

Osano, H., and T. Inoue (1991): "Testing Between Competing Models Of Real Business Cycles," *International Economic Review*, 32(3), 669–688.

Rosen, S. (1985): "Implicit Contracts: A Survey," *Journal of Economic Literature*, 23, 1144–1175.

Stock, J. H., and M. W. Watson (1993): "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica*, 61(4), 783–820.

Wright, R. (1988): "The Observational Implications of Labor Contracts in A Dynamic General Equilibrium Model," *Journal of Labor Economics*, 6, 530–551.

Zeldes, S. (1989): "Consumption And Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, 97(2), 305–346.